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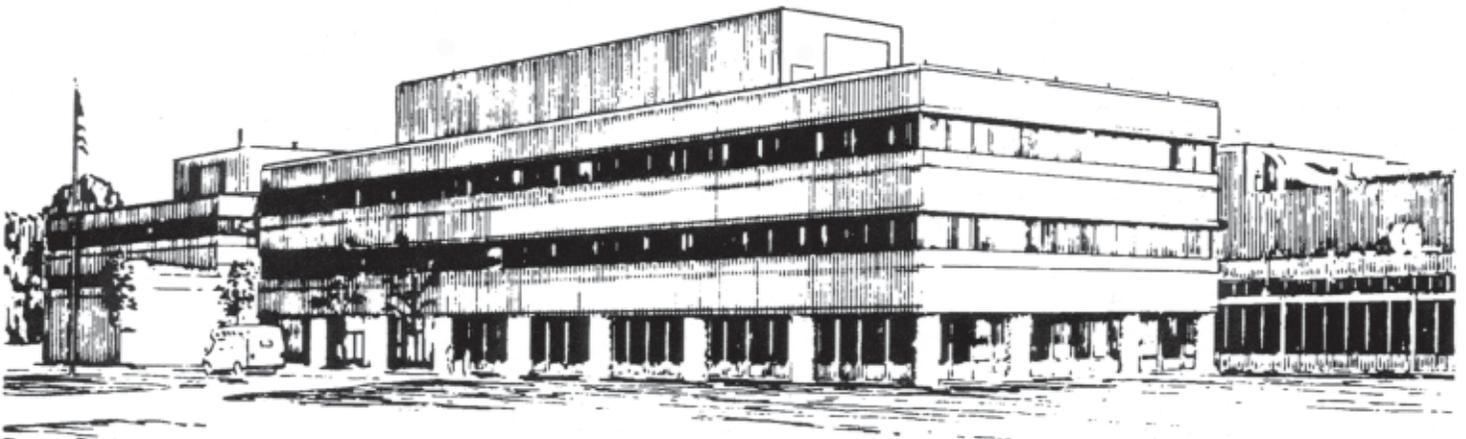
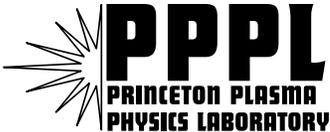
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by Circulating Energetic Ions**

by

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Double-kink fishbone instability caused by circulating energetic ions

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Abstract

The destabilization of double kink modes by the circulating energetic ions in tokamaks with the plasma current having an off-axis maximum is studied. It is shown that the high-frequency fishbone instability [Energetic Particle Mode (EPM)] and the low-frequency (diamagnetic) fishbones are possible for such an equilibrium, their poloidal and toroidal mode numbers being not necessarily equal to unity. A new kind of the EPM instability, “doublet fishbones”, is predicted. This instability is characterized by two frequencies; it can occur in a plasma with a non-monotonic radial profile of the energetic ions when the particle orbit width is less than the width of the region where the mode is localized. It is found that the diamagnetic fishbone branch exists even when the orbit width exceeds the mode width; in this case, however, the instability growth rate is relatively small.

I. INTRODUCTION

Fishbone instability was observed for the first time 20 years ago.¹ It was characterized by the dominant mode numbers (both, poloidal, m , and toroidal, n) equal unity and resulted in strong loss of the injected ions. This instability was interpreted as an $m = n = 1$ rigid kink displacement of the plasma core with $r \leq r_s$, where r_s is the radius of the m/n surface. The calculated frequency was equal either to the precessional frequency of the energetic ions, ω_D [in which case the observations were explained by an Energetic Particle Mode (EPM) instability] or to the bulk-ion diamagnetic frequency, ω_{di} .^{2,3} The mentioned instability was driven by the trapped energetic ions. Later a similar instability caused by the circulating ions was observed, and a theory of the destabilization of the $m = n = 1$ rigid kink by circulating ions was suggested.⁴⁻⁶ Recently, experiments on the National Spherical Torus Experiment (NSTX) and a theoretical consideration have shown that fishbone activity can have $m \neq 1$, $n \neq 1$ dominant mode numbers.⁷ On the other hand, theory predicts that when β is so high that the magnetic valley (a minimum of the equilibrium magnetic field) arises in the vicinity of the magnetic axis, the fishbone instabilities tend to be stabilized.⁸⁻¹⁰

The instabilities described above were relevant to plasmas with the monotonic profile of the safety factor, $q(r)$. On the the hand, it is known that regimes with non-monotonic $q(r)$ are characterized by improved plasma confinement and attract a considerable attention. Therefore, it is of interest to consider fishbone instabilities in plasmas with $q(r)$ having an off-axis minimum. A step in this direction was done in Ref.¹¹, where it was shown that trapped energetic ions can destabilize a double kink mode. The considered instability is localized in the region $r_{s1} < r < r_{s2}$, where r_{s1} and r_{s2} are the $q = m/n \equiv q_s$ radii; it is characterized by a “top-hat” radial plasma displacement, $\xi_r(r)$.¹² The destabilization of the double kink by the circulating ions is not studied yet. This issue is addressed in the present work.

As in Refs.^{12,11}, we assume that $q(r)$ has a single off-axis minimum, q_{min} , and the mode width, Δ_m , is small as compared to r_{min} , where r_{min} is determined by $q(r_{min}) = q_{min}$.

The fishbone mode induced by the circulating particles exists due to energetic-ions finite orbit width, which provides a considerable energy exchange between the circulating particles and a kink perturbation.^{5,6} Therefore, an important parameter is the ratio of the orbit width, Δr_b , to the mode width, Δ_m . In this work, we consider two limit cases, $\Delta_m \ll \Delta r_b$ and $\Delta r_b \ll \Delta_m$. In the latter case there are two characteristic frequencies, $\omega_1 \sim k_{\parallel}^{inner} v_{\parallel}$ and $\omega_2 \sim k_{\parallel}^{outer} v_{\parallel}$, where $k_{\parallel 1}$ and $k_{\parallel 2}$ are characteristic longitudinal wave numbers near the inner edge and outer edge of the region where the mode is localized, respectively, v_{\parallel} is the velocity of the energetic ions along the magnetic field. A possibility of a “doublet” instability, i.e., the instability with $\omega \sim \omega_1$ and $\omega \sim \omega_2$ of a mode with given m, n is considered in the work. The analysis is carried out in the assumption that the energetic ions have “standard” orbits with $\Delta r_b \ll r$.

The structure of the work is as follows. In the Sec. II A the equations describing both the high frequency fishbone branch and the low frequency branch in a plasma with the non-monotonic profile of the safety factor are formulated. These equations are analyzed for the cases of $\Delta_m \ll \Delta r_b$ and $\Delta_m \gg \Delta r_b$, in Sec. II B and Sec. II C, respectively. Section III contains the summary of the obtained results and a discussion of a possible relevance of the predicted “doublet” instability to experimental observations on the ASDEX-U tokamak.¹³

II. FISHBONE ANALYSIS

A. Generic equations

Let us assume first that $\omega \gg \omega_{di}$. Then a generic form of the dispersion relation describing fishbone instabilities (and the influence of energetic ions on the ideal MHD instabilities) is

$$\delta I + \delta W_c + \delta W_h = 0, \quad (1)$$

where δW_c is the MHD potential energy of the bulk plasma, δW_h is the energy associated with the response of the energetic ions, and δI is the sum of the kinetic and potential energies in two inertial layers at r_{s1} and r_{s2} given by (cf.¹²)

$$\begin{aligned}
\delta I &= -2\pi^2 \frac{R_0}{m^2} n_0 m_i \int_{layers} r^3 \left(\frac{d\xi_r}{dr} \right)^2 [\omega^2 - k_{\parallel}^2 v_A^2] dr \\
&= -\frac{i\omega}{2v_A} \frac{B^2}{|m|q_s} (r_{s1}^2 |s_1| + r_{s2}^2 |s_2|) \xi_0^2.
\end{aligned} \tag{2}$$

Here ω is the mode frequency (we assume $\omega > 0$), v_A is the Alfvén velocity, $k_{\parallel} = (mq^{-1} - n)/R_0$ is the longitudinal wave number, R_0 is the radius of the magnetic axis, ξ_0 is the amplitude of the radial component of the MHD displacement. The latter is taken in the form: $\vec{\xi} = \vec{\xi}_0(r) \exp[i(m\theta - n\phi - \omega t)]$, with $\vec{\xi}_0 = (\xi_{0r}, \xi_{0\theta})$, $\xi_{0r}(r) = \xi_0 H(r - r_{min} + \Delta_m/2) H(r_{min} + \Delta_m/2 - r)$, $H(x) = \int_{-\infty}^x \delta(t) dt$, and $\xi_{0\theta}$ is determined by the incompressibility condition $\nabla \cdot \vec{\xi} = 0$.

It follows from Eqs. (1), (2) that the dispersion relation can be written in a form similar to that in the case of the monotonic $q(r)$:

$$i \frac{\omega}{\omega_A} + \lambda_c + \lambda_h = 0, \tag{3}$$

where λ_c and λ_h are the normalized negatives of δW_c and δW_h , respectively, ω_A is the Alfvén frequency. The fast ion response, λ_h , consists of the fluid (adiabatic) part, and the kinetic part: $\lambda_h = \lambda_{hf} + \lambda_k$.

The magnitudes in Eq. (3) can be written as follows (we put $r_{s1} \approx r_{s2} \approx r_{min}$, which is justified due to the assumption $\Delta_m \ll r_s$):^{11,12,14}

$$\begin{aligned}
\omega_A &= |m|(|s_1| + |s_2|) \frac{v_A}{q_s R_0}; \\
\lambda_c &= L\alpha^2 \left(1 - \frac{nq_{min}}{m}\right)^{1/2} - \frac{8}{15}(m^2 - 1) \left(1 - \frac{nq_{min}}{m}\right)^2 \\
&\quad - \frac{8\pi r p'}{B_0^2} (1 - q_{min}^2) - \lambda_{comp},
\end{aligned} \tag{5}$$

where $\alpha = -8\pi R_0 p' q^2 / B_0^2$, $p' \equiv dp/dr$, p is the plasma pressure, L is a number that encapsulates the information regarding coupling to toroidally induced sideband displacements,¹² λ_{comp} is the stabilizing contribution associated with perpendicular compression,¹⁵

$$\lambda_{hf} = -\frac{\pi^2 m_{\alpha} q_s}{\xi_0^2 (|s_1| + |s_2|)^2 r_{min}^2 B_0^2} \sum_{\sigma} \int dP_{\phi} \int v^5 dv \int \frac{\partial F_{\alpha}}{\partial P_{\phi}} \tau_b \langle \xi_{0r}^2 r \cos \theta \rangle d\Lambda; \tag{6}$$

$$\lambda_k = \frac{2\pi^2 R_0 m_\alpha q_s^2}{\omega_{c\alpha} r_{min}^2 \xi_0^2 B_0^2 (|s_1| + |s_2|)^2} \sum_\sigma \int v^3 dv \int dP_\phi \int d\Lambda \tau_b \frac{\partial F_\alpha}{\partial \mathcal{E}} \frac{\omega - n\omega_{* \alpha}}{\omega - k_\parallel v_\parallel} \times \left| \left\langle \left(\frac{v_\perp^2}{2} + v_\parallel^2 \right) \vec{\xi} \cdot \vec{\kappa} \exp[i(\omega - k_\parallel v_\parallel)t] \right\rangle \right|^2, \quad (7)$$

where $\mathcal{E} = m_\alpha v^2/2$ is the particle energy, $\Lambda \equiv \mu B_0/\mathcal{E}$, $P_\phi = m_\alpha v_\parallel R + (e/c)\psi$ is the canonical angular momentum, ψ is the poloidal magnetic flux, R is the distance from the major axis of the torus, $\omega_{* \alpha} = (\partial F_\alpha/\partial P_\phi)(\partial F_\alpha/\partial \mathcal{E})^{-1}$, τ_b is the particle transit time, $\langle \dots \rangle$ denotes the orbit averaging, $\vec{\kappa} = B^{-2} \nabla_\perp (B^2/2 + 4\pi p)$ is the field line curvature, $\sigma \equiv v_\parallel/|v_\parallel|$, v_\parallel and v_\perp are the particle velocity along and across the magnetic field, respectively.

Taking into account that λ_c does not depend on ω and assuming $\omega = \omega_0 + i\gamma$ with $\gamma \ll \omega_0$, we can split Eq. (3) into two equations representing the real and imaginary parts of Eq. (3) as follows (the subscript ‘‘0’’ is omitted):

$$\frac{\gamma}{\omega_A} = (\lambda_c + \text{Re } \lambda_h) \left(1 + \omega_A \frac{\partial \text{Im } \lambda_h}{\partial \omega} \right)^{-1}, \quad (8)$$

$$-\frac{\gamma \omega_A}{\omega} \frac{\partial \text{Re } \lambda_h}{\partial \omega} = 1 + \frac{\omega_A \text{Im } \lambda_h}{\omega}. \quad (9)$$

Equations (8), (9) impose certain restrictions on λ_h . In particular, Eq. (8) with $\gamma \geq 0$ and $\omega_A \partial \text{Im } \lambda_h / \partial \omega > -1$ is satisfied for a plasma on/below the ideal MHD stability limit ($\lambda_c \leq 0$) only when $\text{Re } \lambda_h \geq 0$. The first term in the RHS of Eq. (9) describes the continuum damping of the mode, whereas the second term drives the instability provided that $\text{Im } \lambda_h < 0$. It follows from Eq. (9) that $\partial \text{Re } \lambda_h / \partial \omega > 0$ for the instability to occur, which implies that the mode energy is negative, and thus Eq. (3) describes an EPM instability.

The sign of $\partial \text{Im } \lambda_h / \partial \omega$ depends on the sign of dp_α/dr (with p_α the fast ion pressure) when the inhomogeneity term dominates in Eq. (7). Therefore, the physics of the destabilization of a double kink mode by the energetic ions with a non-monotonic $p_\alpha(r)$ and $|\omega_A \partial \text{Im } \lambda_h / \partial \omega| \sim 1$ is more complicated than that in the case of a monotonic $p_\alpha(r)$.

When $\omega \sim \omega_{di}$, the diamagnetic effects should to be taken into account. A corresponding dispersion relation for the monotonic $q(r)$ has been obtained in Ref.³ One can see that

it has a similar form for the non-monotonic $q(r)$ when $\omega_{di}(r_{s1}) = \omega_{di}(r_{s2})$, which can be justified in the considered case of $\Delta_m \ll r_{min}$:

$$[-\omega(\omega - \omega_{di})]^{1/2} = \omega_A[\lambda_c + \lambda_{hf} + \lambda_k(\omega)], \quad (10)$$

where $\lambda_c + \lambda_{hf} + Re\lambda_k \geq 0$. The low-frequency (diamagnetic) fishbone branch is described by Eq. (10) in the case of

$$\omega_A[\lambda_c + \lambda_h(\omega = 0)] \ll 0.5\omega_{di}, \quad (11)$$

i.e., when the kink/tearing instability is stabilized by the plasma diamagnetism. Then Eq. (10) has the following fishbone solution obtained in the assumption $|Im \lambda_h| \ll (\lambda_c + Re \lambda_h)$:

$$\omega \approx \omega_{di}, \quad \gamma = -2\omega_{di}[\lambda_c + Re \lambda_h(\omega_{di})]Im \lambda_h(\omega_{di}). \quad (12)$$

It is clear that the energy associated with the energetic ions considerably depends on the $q(r)$ profile. When $q(r)$ is non-monotonic, there are two cases with different physics, $\Delta_m > \Delta r_b$ and $\Delta_m < \Delta r_b$, which will be considered below.

B. The case of large orbit width

Let us first carry out an analysis in the assumption of $\Delta_m \ll \Delta r_b$. We consider a plasma with a population of the energetic ions consisting only of the well-circulating particles. Then the equilibrium distribution function F_α can be approximated as follows:

$$F_\alpha = \frac{\sqrt{2}m_\alpha^{3/2}}{\pi\mathcal{E}_\alpha} p_\alpha(\bar{r}) H(\mathcal{E}_\alpha - \mathcal{E}) \mathcal{E}^{-3/2} \delta(\Lambda), \quad (13)$$

where $p_\alpha(\bar{r})$ is the beam particle pressure [$p_\alpha(\bar{r}) = \int d^3v \mathcal{E} F_\alpha$], \bar{r} is the orbit averaged radial coordinate of the particle. In addition, in this case we can put $v_{||}(t) = const$, $v_{\perp}(t) = const$. This enables us to write $\vec{\xi} = \vec{\xi}_0 \exp[-i(\omega - k_{||}v_{||})t]$. Using this equation and omitting terms odd in θ we have:

$$\langle \vec{\xi} \cdot \vec{\kappa} \exp[i(\omega - k_{||}v_{||})t] \rangle = -\frac{1}{R_0} \langle \xi_{0r} \cos \theta \rangle = \frac{\xi_0}{\pi R_0} (\sin \theta_{*2} - \sin \theta_{*1}), \quad (14)$$

where $\theta_{*1,2}$ are the poloidal angles at which a particle intersects the mode (see Fig. 1a), $\cos \theta_* = (\bar{r} - r_{min} \pm \Delta_m/2)/\Delta_b$, with $\Delta_b = \Delta r_b/2$ (we used $r(\theta) = \bar{r} - \Delta_b \cos \theta$). Due to the small width of the mode we take into account only those particles, which intersect the mode region twice. The radial coordinate of these particles varies from $r > r_{s2}$ to $r < r_{s1}$ during the orbital motion, therefore for them

$$(r_{min} - \bar{r})^2 < (\Delta_b - 0.5\Delta_m)^2. \quad (15)$$

Then we can assume

$$|r - r_{min}|\Delta_m \ll \Delta_b^2 - (r - r_{min})^2, \quad (16)$$

which results in the following approximation of Eq. (14)

$$\langle \vec{\xi} \cdot \vec{\kappa} \exp[i(\omega - k_{\parallel}v_{\parallel})t] \rangle \approx \frac{\Delta_m}{\pi\Delta_b} \cot \hat{\theta}, \quad (17)$$

where $\hat{\theta}$ is defined by $\cos \hat{\theta} = (\bar{r} - r_{min})/\Delta_b$.

Putting Eqs. (17), (13) into Eq. (7), we obtain in the limit $\omega \ll n\omega_{*\alpha}$:

$$\lambda_k = -\frac{m}{2(|s_1| + |s_2|)^2} \left(\frac{\Delta_m}{r_{min}}\right)^2 \omega_{c\alpha} \int_0^1 \frac{xdx}{\omega - \sigma k_{\parallel}(\bar{r})v_{\alpha}x} \int \cot^2 \hat{\theta}(x, \bar{r}) \frac{d\beta_{\alpha}}{d\bar{r}} d\bar{r}, \quad (18)$$

where $x = v/v_{\alpha}$. The domain of two-dimensional integration in Eq. (18) is determined by Eq. (15) and $\Delta_m/(2\Delta_{b\alpha}) \leq x$ with $\Delta_{\alpha} = \Delta(v = v_{\alpha})$, $v_{\alpha} = \sqrt{2\mathcal{E}_{\alpha}/m_{\alpha}}$ [the latter condition follows from $\Delta_b(v) > \Delta_m$].

The imaginary part of λ_k is associated with the resonance condition $\omega = k_{\parallel}v_{\parallel}$. Therefore, $Im \lambda_k \neq 0$ only when $\sigma k_{\parallel}(\bar{r}) > 0$. One can see that $k_{\parallel} > 0$ in the region $r_{s1} < r < r_{s2}$, where it has a maximum (for $m > 0$), and $k_{\parallel} < 0$ outside this region: this conclusion follows from the facts that $k_{\parallel}(r_{s1}) = k_{\parallel}(r_{s2}) = 0$, and $dk_{\parallel}/dr = 0$, $d^2q/dr^2 > 0$ at $r = r_{min}$. This implies that only the particles with \bar{r} located inside the mode region interact with the mode when $v_{\parallel} > 0$. However, for these particles $\cos \hat{\theta} \ll 1$ (because of the assumption $\Delta_m \ll \Delta r_b$). We conclude from here that strong energy exchange between the particles and the mode is possible only when $v_{\parallel} < 0$, in which case $\cos \hat{\theta} \sim 1$ due to the particles in the regions $r < r_{s1}$, $r > r_{s2}$.

In order to calculate the integrals in Eq. (18) analytically, we have to make some approximation for k_{\parallel} . Taking into account that $k_{\parallel}(r_{s1}) = k_{\parallel}(r_{s2}) = 0$ and $dk_{\parallel}/dr = 0$ at $r = r_{min}$, we approximate k_{\parallel} as follows:

$$k_{\parallel} = k_{\parallel min} \frac{4(r - r_{min} + \Delta_m/2)(r_{min} + \Delta_m/2 - r)}{\Delta_m^2}. \quad (19)$$

Equation (19) is simplified in the region which mainly contributes to the integral:

$$k_{\parallel} \approx -k_{\parallel min} \frac{4(r - r_{min})^2}{\Delta_m^2}. \quad (20)$$

Using Eqs. (20), (18), (6), we obtain:

$$\begin{aligned} \lambda_k &= -\frac{mH(-\sigma)}{8(|s_1| + |s_2|)^2} \left(\frac{\Delta_m}{r_{min}}\right)^2 \frac{\omega_{c\alpha}}{k_{\parallel min} v_{\alpha}} \left(\frac{\Delta_m}{\Delta_{\alpha}}\right)^2 \Delta_{b\alpha} \frac{d\beta_{\alpha}}{d\bar{r}} \Big|_{r_{min}} \\ &\times \int_0^1 x dx \int_{\frac{\Delta_m}{2\Delta_{\alpha}} - x}^{x - \frac{\Delta_m}{2\Delta_{\alpha}}} \frac{u^2 du}{(\tilde{\omega} - u^2 x)(x^2 - u^2)} \approx \frac{mH(-v_{\parallel})}{24(|s_1| + |s_2|)^2} \frac{\Delta_m^4}{r_{min}^2 \Delta_{\alpha}^2} \\ &\times \left(\frac{\Delta_{\alpha}}{k_{\parallel} \rho_{\alpha}} \frac{d\beta_{\alpha}}{d\bar{r}}\right) \Big|_{r_{min}} \left[\ln \frac{1 + \sqrt{\tilde{\omega}}}{1 - \sqrt{\tilde{\omega}}} \ln \frac{1 + \sqrt{\tilde{\omega}}}{\sqrt{\tilde{\omega}} - 1} + \ln \frac{4\Delta_{\alpha}}{\Delta_m} \ln \left(1 - \frac{1}{\tilde{\omega}}\right) \right], \end{aligned} \quad (21)$$

$$\lambda_{hf} = -\frac{2q_s^2 \Delta_m}{(|s_1| + |s_2|)^2} \frac{R}{r_{min}} \frac{d\beta_{\alpha}}{dr} \Big|_{r_{min}} \int_0^1 dx \int_{\frac{\Delta_m}{\Delta_{b\alpha}} - x}^{x - \frac{\Delta_m}{\Delta_{b\alpha}}} \frac{udu}{\sqrt{x^2 - u^2}} = 0, \quad (22)$$

where $\tilde{\omega} \equiv (\omega/k_{\parallel min} v_{\alpha})(\Delta_m/\Delta_{b\alpha})^2$.

An EPM instability has the frequency $\omega \lesssim k_{\parallel} v_{\parallel}$, i.e., $\tilde{\omega} \lesssim 1$. Therefore, we can conclude from Eq. (21) that, when the radial profile of the energetic ions is monotonically decreasing, $\text{Re } \lambda_k < 0$ (for all reasonable values of $\Delta_{b\alpha}/\Delta_m$) and $\partial \text{Im } \lambda_k / \partial \omega > 0$. This means that a necessary condition for the existence of the EPM mode, $\text{Re } \lambda_h > 0$ [see Eq. (8)] is not satisfied and, thus, the EPM fishbone mode is absent.

When $\omega \sim \omega_{di}$, Eq. (10) should be used. Its fishbone solution is given by Eq. (12). After calculation of $\text{Im } \lambda_k$ we obtain:

$$\begin{aligned} \gamma &= -\frac{\pi m q_s}{12(|s_1| + |s_2|)^2} [\lambda_c + \text{Re } \lambda_k(\omega = \omega_{di})] \left(\frac{1}{k_{\parallel}} \frac{d\beta_{\alpha}}{dr}\right) \Big|_{\bar{r}=r_{min}} \\ &\times \frac{\omega_A^2}{\omega_{di}} \frac{\Delta_m^4}{r_{min}^2 \Delta_{b\alpha}^2} \ln \frac{4\Delta_{b\alpha}}{\Delta_m}. \end{aligned} \quad (23)$$

This equation describes a low-frequency (diamagnetic) fishbone instability. The instability growth rate is rather small being proportional to small parameters Δ_m^2/r_{min}^2 and Δ_m^2/Δ_b^2 .

C. The case of small orbit width

Now we assume that the energetic ion orbit width is small, $\Delta_b \ll \Delta_m$, but not vanishing: Finite magnitude of the orbit width plays an important role by providing the energy exchange between the well-circulating particles and a periodic perturbation with a “top-hat” $\xi_0(r)$; only particles crossing the region of the mode localization during their orbital motion contribute to $\text{Im } \lambda_k$. Because of the assumption $\Delta_b \ll \Delta_m$, the energetic ions crossing r_{s1} do not reach r_{s2} and vice versa, see Fig. 1b. This implies that the energetic ion response is actually the response calculated for the case of the $m = n = 1$ rigid kink displacement in Ref.⁵, but with two rational surfaces. Therefore, we can easily generalize the result of Ref.⁵. We have:

$$\lambda_k = \sum_{i=1,2} \lambda_{ki} = \sum_{i=1,2} \frac{1}{3} \frac{R_0}{r_{si}} \frac{q_s^2}{(|s_1| + |s_2|)^2} \left[-\frac{\Delta r_{b\alpha}}{|s|} \frac{d\beta_\alpha}{dr} \right]_{r_{si}} F\left(\frac{\omega}{\omega_{si}}\right), \quad (24)$$

where

$$F(x) \equiv \frac{1}{\pi} \left\{ 10x - 8x^{3/2} \left[\tan^{-1} \frac{1}{\sqrt{x}} + \tanh^{-1} \frac{1}{\sqrt{x}} \right] + (1 + 3x^2) \ln \frac{1+x}{x-1} \right\}, \quad (25)$$

$\omega_{si} = |s_i| v_{\parallel\alpha}^2 / (\omega_c R_0 r_{si})$. Note that $\omega_{si} \sim \omega_D$, where ω_D is the precessional frequency of the well-trapped particles, when $s \sim q/2$.

It follows from Eqs. (24), (25) that $\text{Re } \lambda_k > 0$ for $x > 0$ and the monotonically decreasing profile of the energetic ions. Therefore, according to Eqs. (8), (9), in this case the system can be on the margin of stability ($\gamma = 0$) only when a plasma is below the ideal MHD limit ($\lambda_c < 0$). When $\lambda_c > 0$, Eq. (8) predicts $\gamma > 0$.

Below we carry out a more detailed stability analysis. We proceed from Eq. (3) valid for the arbitrary ratio of γ/ω . Assuming $s_1 = s_2$, $d\beta_\alpha/dr|_{r_{s1}} = d\beta_\alpha/dr|_{r_{s2}}$, and $v_A(r_{s1}) = v_A(r_{s2})$, we write this equation as follows:

$$0 = D(\Omega) = -i\Omega - \tilde{\lambda}_c - \pi_\alpha F(\Omega), \quad (26)$$

where $\tilde{\lambda}_c = \lambda_c \omega_A / \omega_{s1}$, $\Omega = \omega / \omega_{s1}$,

$$\pi_\alpha = -\frac{2}{3} \frac{m(m/n)^2}{|s|^3} \frac{v_A}{v_\alpha} R_0 \frac{d\beta_\alpha}{dr} \Big|_{r_s}. \quad (27)$$

The unstable solutions of the dispersion relation can be examined by a Nyquist analysis, see, e.g.¹⁶ Consider a contour in the Ω plane encircling the upper half plane, as shown in Fig. 2a. Its image under the map $D(\Omega)$ is presented in Fig. 2b. In this figure, when π_α exceeds a certain magnitude, $D(\Omega)$ encircles the coordinate origin, which corresponds to an unstable solution. The frequencies and the growth rates can be calculated from Eqs. (8) (9). The results presented in Fig. 3 shows that there are solutions with $\Omega \lesssim 1$ and, thus, an EPM instability is possible.

The found fishbone mode exists due to the resonance $\omega = k_{\parallel} v_{\parallel}$, which leads to one characteristic frequency $[\omega_s \sim k'_{\parallel}(r_s) \Delta_b v_\alpha]$ when $k'_{\parallel}(r_{s1}) = k'_{\parallel}(r_{s2})$. The latter is justified for $k_{\parallel}(r)$ symmetric with respect to r_{min} in the region $r_{s1} < r < r_{s2}$. However, in general, $s(r_{s1}) \neq s(r_{s2})$ for a double kink mode with the finite width. For this reason, there are two, rather than one, characteristic frequencies. Therefore, one can expect that two fishbone instabilities with the frequencies of the order $\omega_{s1} \equiv \omega_s(r_{s1})$ and $\omega_{s2} \equiv \omega_s(r_{s2})$ and the same mode numbers (m, n) will exist simultaneously under some conditions. Below we examine this possibility. We proceed from Eq. (3) written as

$$0 = D(\Omega) = -i\Omega - \tilde{\lambda}_c - \pi_{\alpha 1} F(\Omega) - \pi_{\alpha 2} F(|s_1|\Omega/|s_2|), \quad (28)$$

where

$$\pi_{\alpha i} = -\frac{8}{3} \frac{m(m/n)^2}{(|s_1| + |s_2|)^2} \left(\frac{v_A R_0}{v_\alpha} \frac{d\beta_\alpha}{s dr} \right)_{r=r_{si}}. \quad (29)$$

The results of the Nyquist analysis for two particular cases are presented in Fig. 4. We observe that in both cases the curves encircle the origin of coordinate twice. This means that two instabilities co-exist, which we will refer to as a “doublet” instability. Their growth rates and the frequencies calculated as a function of $\pi_{\alpha 2}$ for $|s_1/s_2| = 0.6$ and $\pi_{\alpha 1} = -2.5$ with the use of Eqs. (8), (9) are shown in Fig. 5. We observe that, first, there is no threshold with respect to $\pi_{\alpha 2}$ and, second, two instabilities with different frequencies exist for $\pi_{\alpha 2}$ in a certain interval (shaded region in Fig. 5). It is clear that one of the instabilities (with a lower frequency) arises mainly due to particles crossing r_{s1} when $\pi_{\alpha 2}$ is close to zero. Another instability, characterized by a higher frequency, arises when $\pi_{\alpha 2}$

exceeds the threshold magnitude $\pi_{\alpha 2}^{crit} \approx 2.4$. When $\pi_{\alpha 2}$ exceeds a certain magnitude, low frequency instability disappears. However, a more rigorous analysis is required to find the region where it disappears because the shown picture is based on the equations valid for $\gamma \ll \omega \lesssim \omega_s$. The Nyquist technique leads to $2.32 < \pi_{\alpha 2} < 2.45$. Thus, the region is rather narrow. On the other hand, calculations show that the region of the existence of the “doublet” instability is larger when a plasma is above the ideal MHD stability limit ($\lambda_c > 0$). For instance, we obtain $2.25 < \pi_{\alpha 2} < 2.47$ for $\lambda_c = 0.1$ and the same other parameters.

Let us proceed to the low frequency range, $\omega \sim \omega_{di} \ll \omega_s$. Using Eqs. (24), (12) we obtain:

$$\gamma = -\frac{4}{3} \frac{\omega_A^2}{\omega_{di}} (\lambda_c + \lambda_{hf}) \frac{q_s^2}{(|s_1| + |s_2|) |s_1 s_2| r_{min}} \frac{R_0}{r_{min}} \Delta_{b\alpha} \left. \frac{d\beta_\alpha}{dr} \right|_{r_{min}}. \quad (30)$$

Comparing Eq. (30) with Eq. (23) we conclude that the growth rate of the instability caused by the energetic ions with the narrow orbit width considerably exceeds the one associated with the wide-orbit-width particles.

III. SUMMARY AND CONCLUSIONS

In summary, we have developed a theory of the destabilization of the double kink mode by the circulating energetic ions through the resonance $\omega = k_{\parallel} v_{\parallel}$. It shows that both the EPM instability and low frequency fishbone instability ($\omega \approx \omega_{di}$) can be driven by the circulating beam ions in plasmas with the non-monotonic safety factor. The instability is localized between two rational surfaces, r_{s1} and r_{s2} , with the same safety factor, $q_s(r_{s1}) = q_s(r_{s2}) = m/n$. A key parameter, which affects the instability growth rate, is the ratio of the orbit width of the energetic ions, Δr_b , to the mode width, Δ_m . The instabilities are most strong when $\Delta r_b \ll \Delta_m$. In the contrary case, $\Delta r_b \gg \Delta_m$, the EPM instability is absent in plasmas with the $d\beta_\alpha/dr < 0$.

A new kind of the instability, which we refer to as “doublet” fishbones, is predicted. This instability is characterized by two frequencies and two growth rates, although it is

relevant to the same double kink mode. The ratio of the frequencies is about the ratio of the shears at two rational surfaces where the instability is localized. The instability can occur when the radial profile of the energetic ions has an off-axis maximum inside the region of the mode localization. The existence of the “double” fishbones is a consequence of the fact that there are two, rather than one, characteristic frequencies in the case of a non-monotonic $q(r)$: $\omega_1 = k_{\parallel}^{inner} v_{\parallel}$ and $\omega_2 = k_{\parallel}^{outer} v_{\parallel}$, where k_{\parallel}^{inner} and k_{\parallel}^{outer} are the characteristic longitudinal wave numbers relevant to the inner edge and outer edge of a double kink mode.

It seems possible that the “doublet” instability was observed in an ASDEX-U experiment reported in Ref.¹⁷: In the mentioned experiment an $m = n = 1$ mode with $f_1 \sim 15$ kHz and $f_2 \sim 20$ kHz (in the laboratory frame) was destabilized during tangential Neutral Beam Injection (NBI); The NBI power was deposited at about a half the plasma radius and, thus, $\beta_{\alpha}(r)$ was non-monotonic; there were two $q = 1$ surfaces due to off-axis NBI current drive. Note that the observed doublet frequencies were comparable with the fishbone frequency during the radial injection,¹⁷ which is consistent with our theory as $\omega_s \sim \omega_D$. Moreover, observation of a continuous mode during tangential injection instead of bursting fishbones during the radial injection can be explained by weak redistribution of the circulating particles crossing r_{s1} and r_{s2} (in contrast to the resonant expulsion of the trapped particles). Of course, a more detailed analysis is required for the reliable interpretation of the experimental results.

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FIGURES

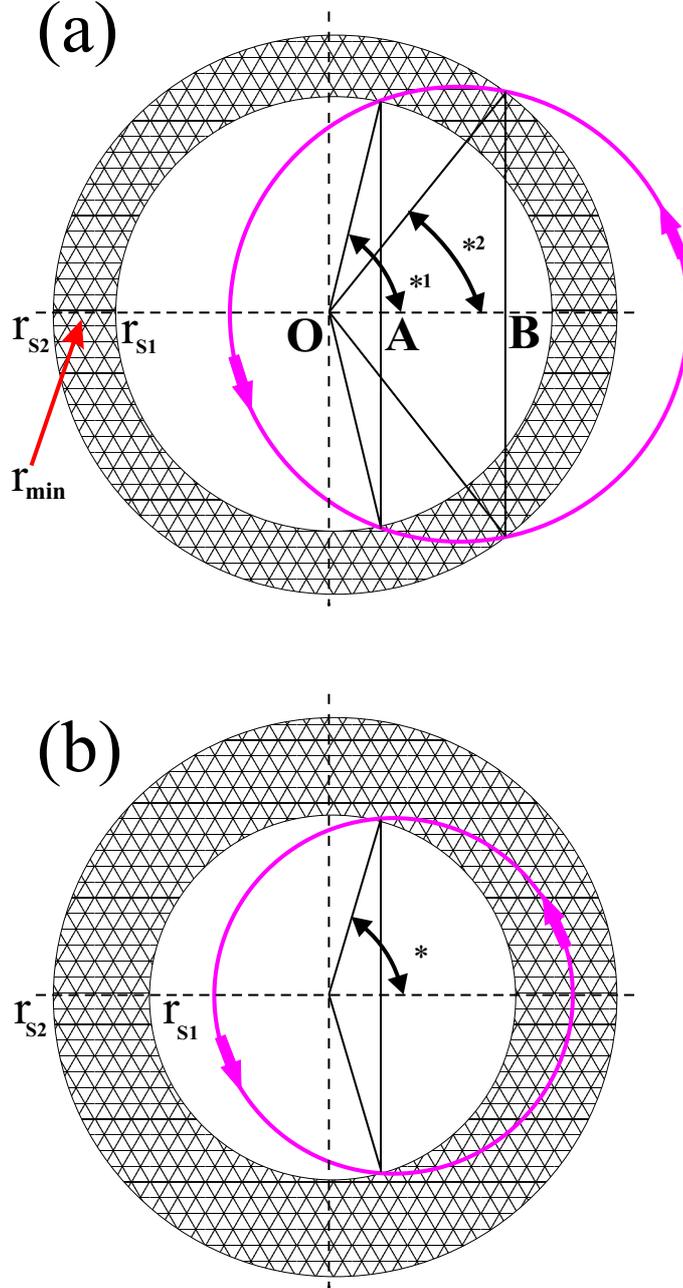


FIG. 1. A particle orbit crossing the region where a double kink mode is localized (shaded region): a, the orbit width exceeds the mode width; b, the mode width exceeds the orbit width. Notations: r_{s1} and r_{s2} are two rational surfaces with the same $q(r)$, r_{min} is the radius where $q = q_{min}$, OA and OB are the cosines of the angles θ_{*1} and θ_{*2} at which a particle crosses the edges of the mode localization region.

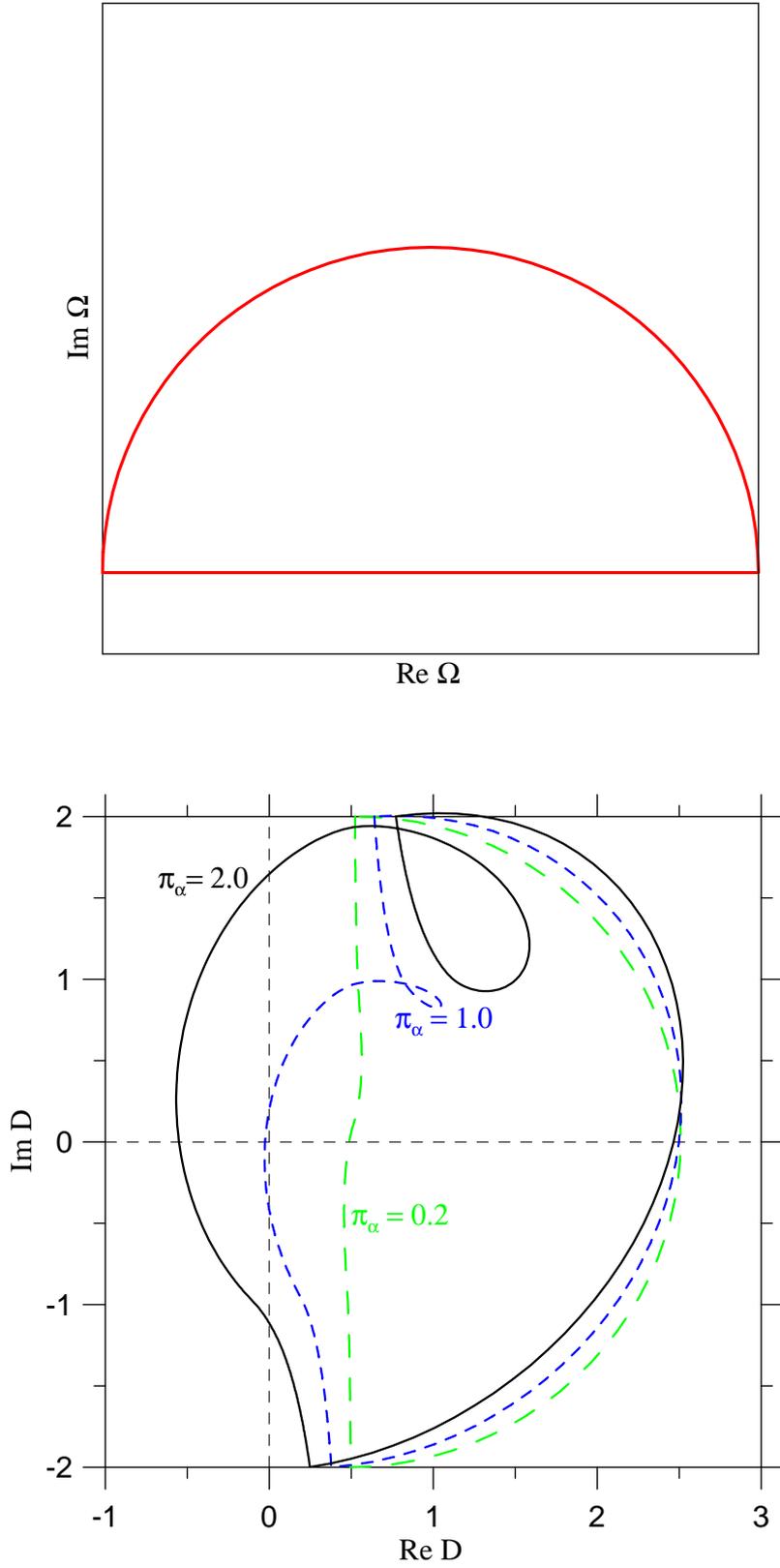


FIG. 2. Nyquist contour in the plane $(\text{Re } \Omega, \text{Im } \Omega)$ and its map in the plane $(\text{Re } D, \text{Im } D)$ when $s_1 = s_2$, $\lambda_c = -0.5$ for various π_α . Notations: $\Omega = \omega/\omega_{s1}$, $\omega_{s1} = |s_1|v_{\parallel\alpha}^2/(\omega_c R_0 r_s)$, $\pi_\alpha \propto -d\beta_\alpha/dr$

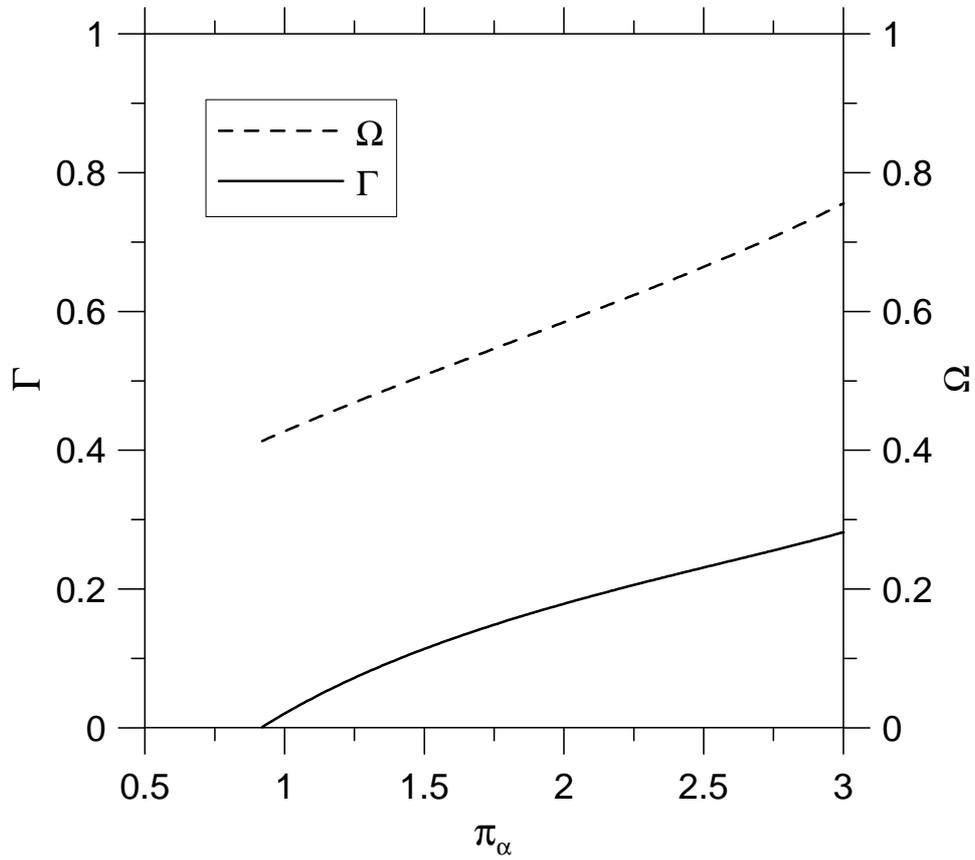


FIG. 3. Normalized growth rate, $\Gamma = \gamma/\omega_{s1}$, and the mode frequency, $\Omega = \omega/\omega_{s1}$, of the EPM fishbone instability in a plasma with a monotonic $\beta_\alpha(r)$ and $\tilde{\lambda}_c = -0.1$.

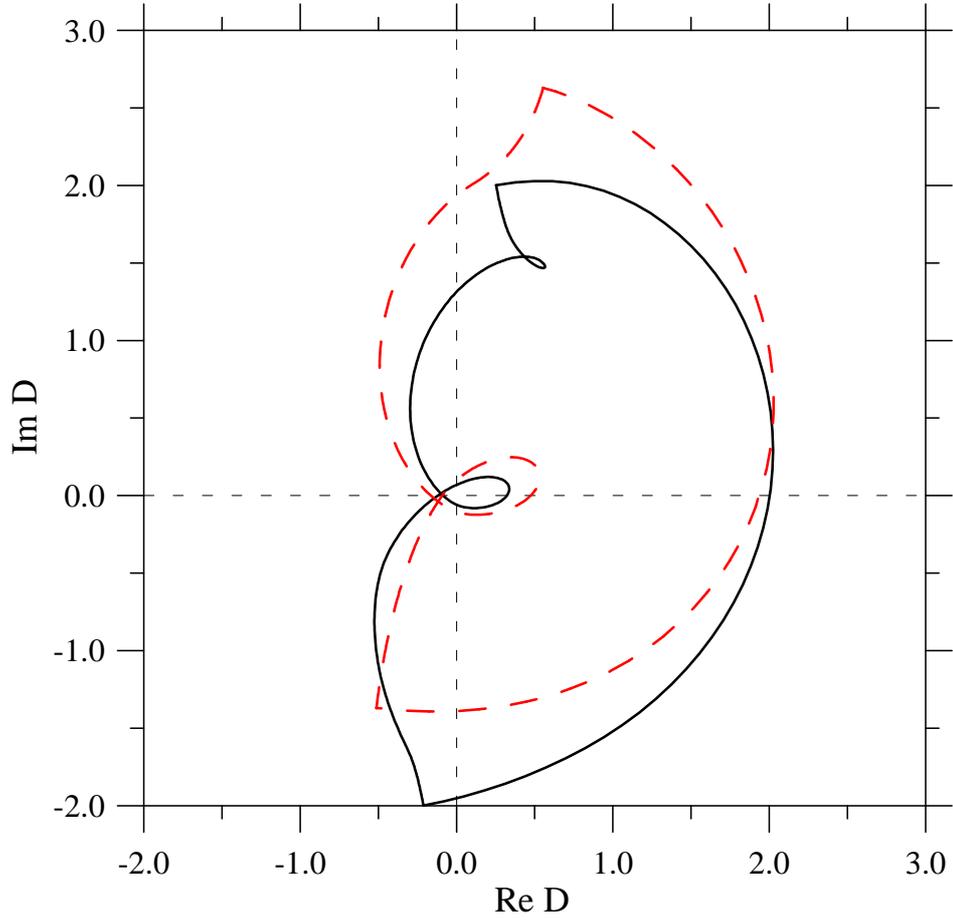


FIG. 4. Map of a Nyquist contour in the case of “doublet” instability for $\tilde{\lambda}_c = -0.01$: solid line, $s_1/s_2 = 0.6$, $\pi_{\alpha_1} = -2.5$, $\pi_{\alpha_2} = 2.4$; dotted line, $s_1/s_2 = 0.2$, $\pi_{\alpha_1} = -1.5$, $\pi_{\alpha_2} = 1.35$.

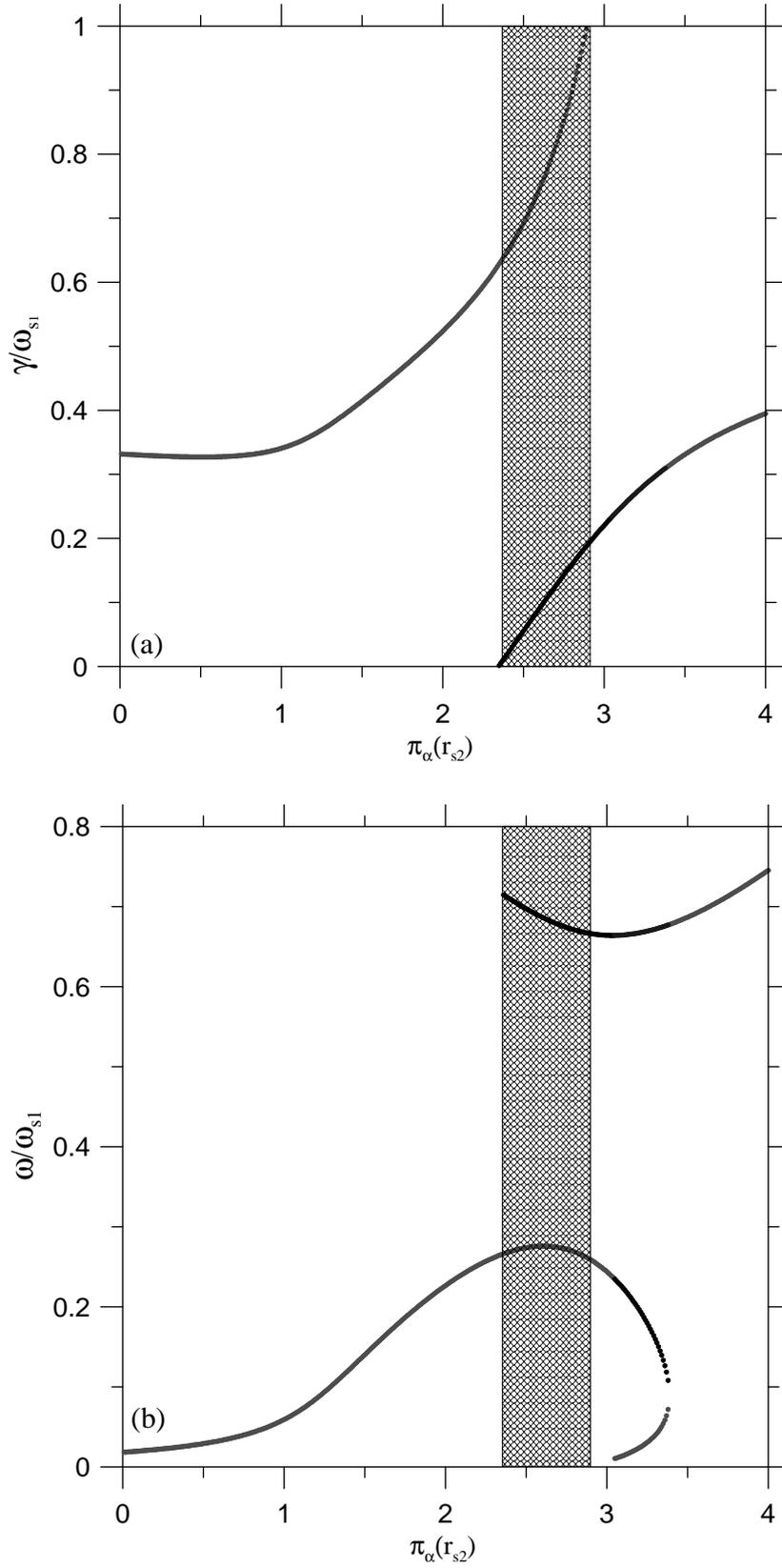


FIG. 5. The EPM fishbone instability in a plasma with a non-monotonic $\beta_\alpha(r)$ for $s_1/s_2 = 0.6$, $\pi_{\alpha 1} = -2.5$, and $\tilde{\lambda}_c = -0.01$: a, γ/ω_{s1} versus $\pi_\alpha(r_{s2})$; b, ω/ω_{s1} versus $\pi_\alpha(r_{s2})$. Hatched region corresponds to a “doublet” instability. Its right edge obtained from the condition $\gamma = \omega_{s1}$ is tentative.

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