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The Theory of the Kink Mode during the Vertical Plasma Disruption Events in Tokamaks

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For easy navigation the enumeration in the Table of Contents, and the "(to ToC)" right after the section names are the forward and backward hyperlinks between Table of Contents and the beginning of sections.

Contents

Abstract

This paper explains the locked $m/n = 1/1$ kink mode during the vertical disruption event when the plasma has an electrical contact with the plasma facing conducting surfaces. It is shown that the kink perturbation can be in equilibrium state even with a stable safety factor $q > 1$, if the halo currents, excited by the kink mode, can flow through the conducting structure. This suggests a new explanation of the so-called sideway forces on the tokamak in-vessel components during the disruption event.

1 Introduction. *(to ToC)*

In the occasion of a vertical disruption event in tokamaks [1, 2, 3, 4] a relatively long lasting kink mode perturbation is present sometimes despite the fact, that the well-known stability criterion $q > 1$ is fulfilled (q is the safety factor). Associated with this kink mode a so-called sideway force, acting on the vacuum vessel, was discovered on JET tokamak [5] and then explained by the toroidal asymmetry of the plasma current [6], which is shared with the the vacuum vessel between two toroidally separated spots where the plasma touches the wall [7, 8].

For general stability theory of tokamaks the observation of a locked $m/n = 1/1$ mode when q is greater than 1 represents a puzzle, the resolution of which becomes even more important because of a very unfavorable scaling of the sideway force from small to large machines.

This paper describes the kink mode $m/n = 1/1$ (m, n are poloidal and toroidal wave numbers) in the situation when the plasma touches the conducting wall surface and some of the plasma current, called halo current, can flow in and out the wall in the "wet" spot area where the plasma is in the contact with the wall.

2 Coordinate system and $m/n = 1/1$ helical perturbation *(to ToC)*

In the following we use the quasi-cylindrical coordinate system ρ, ω, φ, with the origin on the axis of the toroidal plasma column $r = R$, $z = 0$, $0 \le \varphi \le 2\pi$

$$r = R - \rho\cos\omega, \quad z = \rho\sin\omega, \quad \rho = a + \xi(\omega,\varphi), \tag{2.1}$$

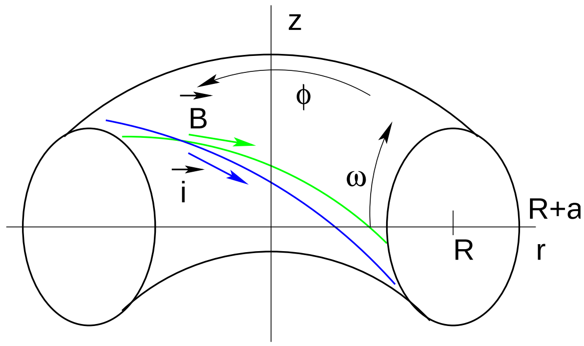


Figure 1. Toroidal plasma column and its angle coordinates

where r, φ, z are the global cylindrical coordinates reflecting the axisymmetry of the unperturbed plasma, a is the plasma minor radius and ξ is the plasma surface perturbation. (For simplicity we consider a plasma with circular cross-section).

The green line in Fig. 1 shows the magnetic field line in the stable case with $q > 1$.

$$q \equiv \frac{aB_\varphi}{RB_\omega}. \tag{2.2}$$

Here B_φ is the toroidal and B_ω is the poloidal magnetic fields.

The blue line in Fig. 1 shows the path of the positive surface current induced by the $m/n = 1/1$ perturbation $\xi = \xi_{11}\cos(\omega - \varphi)$ of the plasma surface. At the other side of the plasma surface the induced current would be negative (opposite to the plasma current). It will be colored red in the following.

2.1 Surface current due to plasma perturbation *(to ToC)*

The value of the surface current $\mathbf{i}(\omega,\varphi)$ in the case of a single harmonic is given by [9, 10]

$$\mu_0\mathbf{i} = -\xi_{11}\frac{2B_\varphi}{R}\cos(\omega-\varphi)\mathbf{e}_\varphi - \frac{1}{R}\xi_{11}\frac{2aB_\varphi}{R^2}\sin(\omega-\varphi)\mathbf{e}_\omega, \quad \mu_0 = 0.4\pi, \tag{2.3}$$

where $\mathbf{e}_\omega, \mathbf{e}_\varphi$ are the unit vectors along coordinates ω, φ. The expression of $\mathbf{i}(\omega,\varphi)$ for arbitrary perturbation is derived in the next section. In general case the surface current can be written in terms of the flow function $I(\omega,\varphi)$

$$\mathbf{i} \equiv \nabla I(\omega,\varphi)\times\mathbf{e}_n = -\frac{1}{a}I'_\omega\mathbf{e}_\varphi + \frac{1}{R}I'_\varphi\mathbf{e}_\omega, \quad \mathbf{e}_n \equiv \mathbf{e}_\omega\times\mathbf{e}_\varphi, \tag{2.4}$$

where $\mathbf{e}_n$ is a unit vector normal to the plasma surface. In our case

$$\mu_0 I(\omega,\varphi) = \xi_{11}\frac{2aB_\varphi}{R}\sin(\omega-\varphi). \tag{2.5}$$

Fig. 2 shows the pattern of the current flow lines $I(\omega,\varphi) = \text{const}$ on the plane φ, ω. The green lines are magnetic field lines. The red lines corresponds to negative surface current (where $\xi(\omega,\varphi)$ is positive), while the blue lines corresponds to positive surface current (where $\xi(\omega,\varphi)$ is negative).

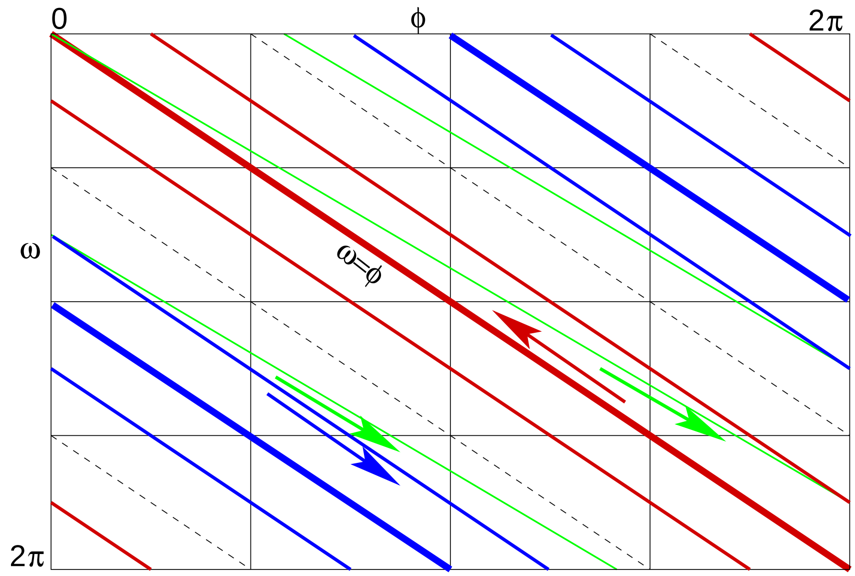


Figure 2. Iso-lines $I(\omega, \varphi) = \text{const}$ for a free boundary plasma.

2.2 Electro-magnetic pressure and the sideway force *(to ToC)*

In the general case, the electromagnetic pressure $p = \mathbf{i} \times \mathbf{B}$ acting on the surface current can be calculated as

$$p = \mathbf{e}_n \cdot ((\nabla I \times \mathbf{e}_n) \times \mathbf{B}) = \mathbf{B}\nabla I. \tag{2.6}$$

In our case the formula reduces to

$$p = 2\xi_{11} \frac{B_\omega B_\varphi}{R\mu_0}(1-q)\cos(\omega - \varphi). \tag{2.7}$$

This pressure can create the "sideway" force $F_x \mathbf{e}_x$ along the horizontal unit vector $\mathbf{e}_x$

$$F_x = \int p(\mathbf{e}_n \cdot \mathbf{e}_x) dS, \tag{2.8}$$

where integration is performed over the entire plasma surface S.

For a single $m/n = 1/1$ perturbation the force can be calculated as

$$F_x = \oint \oint aRp \cos\omega \cos\varphi d\omega d\varphi = \pi I_{pl} B_\varphi (1-q)\xi_{11}, \tag{2.9}$$

where I_{pl} is the plasma bulk current.

2.3 The role of halo currents in the creation of the sideway force *(to ToC)*

So far, there is no wall in our consideration. Electro-magnetic pressure is applied to the plasma surface through the surface current. Eqs.(2.7,2.8) show that at $q > 1$ this pressure is opposite to the plasma displacement in agreement with stability theory.

In the case of marginal stability, $q = 1$, the surface current is driven along the field lines and does not produce a force.

The idea of this paper is to formulate conditions when the surface current $\mathbf{i}(\omega, \varphi)$ on the free plasma surface is directed along the field lines, while in the wetted zone it is partially intercepted by the wall through the halo currents. In this case, the perturbed plasma can be in equilibrium, while the force (2.8) will be applied to the wall as a sideway force.

3 Surface current in presence of halo currents *(to ToC)*

Fig. 3 illustrates the possibility of creating an equilibrium situation for a perturbed plasma even at $q > 1$, if the halo currents are allowed. The rose rectangle covers a wet spot where the plasma touches the wall.

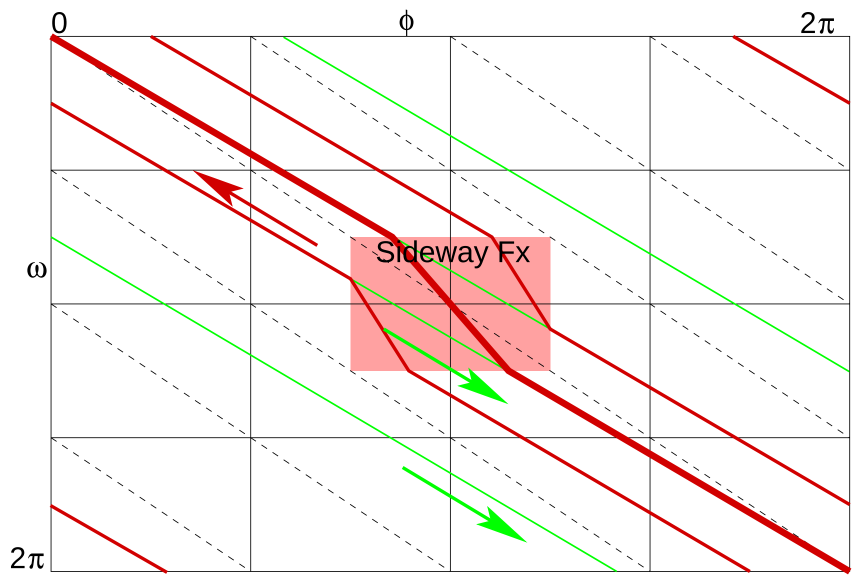


Figure 3. Iso-lines $I(\omega, \varphi) = \text{const}$ for a perturbed plasma with a wet spot on the wall

On the free surface of the plasma the current flows along the field lines and does not suppress the plasma perturbation. In the wet zone, the part perpendicular to $\mathbf{B}$ flows through the wall, while the plasma surface portion flows along the magnetic field lines and does not affect the plasma perturbation. Outside the red flow lines there is no surface current.

The existence of a perturbed equilibrium is the manifestation of a plasma magneto-hydrodynamic instability, which with halo current can be even at $q > 1$.

There is a minimal size of the wet zone when such an equilibrium is possible. In Fig. 3, it is given by a distance between field lines launched from, e.g., $\omega = 0, \varphi = 0$ and $\omega = 2\pi, \varphi = 2\pi$ (the distance between two red lines in Fig. 3). In the case, when the wet spot is related to the helical deformation itself, there is a threshold for the kink perturbation.

With a localized wet spot the kink mode can create not only the sideway force F_x (2.8) but also the toroidally asymmetric vertical force F_z on the in-vessel component

$$F_z = \int p(\mathbf{e}_n \cdot \mathbf{e}_z) dS, \tag{3.1}$$

well detectable in experiments.

The vertical disruption event, associated with a vertical axisymmetric motion, creates the wet zone around the entire toroidal surface at the top of the plasma, as is shown in Fig. 4. In this case, the kink mode becomes unstable with no threshold.

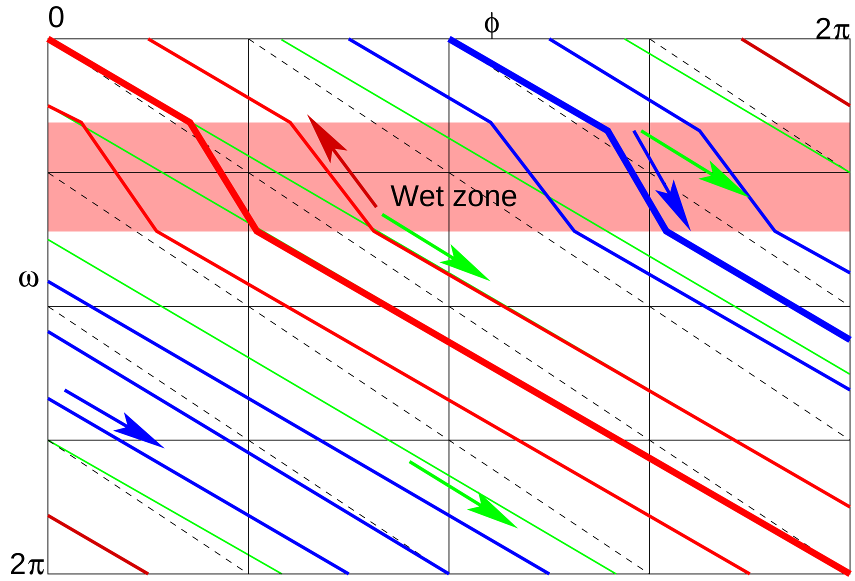


Figure 4. Iso-lines $I(\omega,\varphi)=\mathrm{const}$ for a plasma perturbed by the kink mode for the case when an extended wet zone is created by the vertical plasma motion

The following section describes the formalism of the perturbed plasma equilibrium in the presence of a halo current.

4 Perturbed equilibrium in the presence of a halo current *(to ToC)*

As a first step a plasma with a circular cross-section is considered. This allows one to introduce Fourier harmonics of the plasma displacement as the eigen-perturbations. For noncircular plasma the eigen-perturbations have to be calculated numerically with stability codes.

Also, for analytic consideration a uniform current density distribution is assumed. The $|m|=1$ perturbations are not sensitive to this assumption. For $|m|>1$ the only difference would be the substitution of the left stability boundary for free boundary kink modes by numerically accounting for the non-uniform current.

4.1 Perturbed free boundary equilibria with a uniform current density *(to ToC)*

Coordinate system is ρ,ω,φ. Here, for simulation of toroidal geometry φ is used instead of s, assuming $s=R\varphi$.

Plasma boundary is described by

$$\rho = a + \Re\xi_{mn}e^{im\omega-in\varphi}, \quad \mathbf{e}_p = \mathbf{e}_\omega + \Re\frac{im\xi_{mn}}{a}e^{im\omega-in\varphi}\mathbf{e}_\rho, \quad \mathbf{e}_t = \mathbf{e}_\varphi - \Re\frac{in\xi_{mn}}{a}e^{im\omega-in\varphi}\mathbf{e}_\rho, \tag{4.1}$$

where $\mathbf{e}_p,\mathbf{e}_t$ are the unit vectors on the perturbed surface in the poloidal and toroidal directions. The normal vector $\mathbf{e}_n$ has the form

$$\mathbf{e}_n = \mathbf{e}_p\times\mathbf{e}_t = \mathbf{e}_\rho + \Re\frac{in\xi_{mn}}{a}e^{im\omega-i\in\varphi}\mathbf{e}_\varphi - \Re\frac{im\xi_{mn}}{a}e^{im\omega-i\in\varphi}\mathbf{e}_\omega. \tag{4.2}$$

For helical perturbation the field can be obtained using

$$\mathbf{B}^* \equiv B_\rho\mathbf{e}_\rho + B_\omega\mathbf{e}_\omega - \frac{n\rho}{mR}B_\varphi\mathbf{e}_\omega = \mathbf{e}_\varphi\times\nabla\Psi_{mm} = \Psi'_{mm,\rho}\mathbf{e}_\omega - \Psi'_{mm,\omega}\mathbf{e}_\rho. \tag{4.3}$$

The solution of the equilibrium equation inside the plasma has the form

$$\Psi^i_{mn} = \frac{1}{4}\left(j - \frac{2nB_\varphi}{mR}\right)\left[\rho^2 - 2a\xi_{nm}\frac{\rho^m}{a^m}e^{im\omega - in\varphi}\right]. \tag{4.4}$$

Outside the plasma

$$\begin{aligned}\Psi^e_{mn} &= -\frac{1}{4}\frac{2nB_\varphi}{mR}\left[\rho^2 - 2a\xi_{nm}\frac{\rho^m}{a^m}e^{im\omega - in\varphi}\right] + \frac{1}{4}ja^2\left[\ln\frac{\rho^2}{a^2} - 2\frac{\xi_{nm}}{a}\frac{\rho^m}{a^m}e^{im\omega - in\varphi}\right] \\ &+ \frac{1}{4}j\frac{2\xi_{nm}a}{m}\left(\frac{\rho^m}{a^m} - \frac{a^m}{\rho^m}\right)e^{im\omega - in\varphi}, \\ \Psi^{ext}_{mn} &= -\frac{1}{4}\left[\frac{m-1}{m}j - \frac{2nB_\varphi}{mR}\right]2a\xi_{nm}\frac{\rho^m}{a^m}e^{im\omega - in\varphi}.\end{aligned} \tag{4.5}$$

Current on the plasma surface

$$B^{ext}_\omega = -m\xi_{nm}\frac{1}{2}\left[\frac{m-1}{m}j - \frac{2nB_\varphi}{mR}\right]e^{im\omega - in\varphi}, \quad \mu_0 i_{mn} = \xi_{nm}\left[(m-1)j - \frac{2nB_\varphi}{R}\right]e^{im\omega - in\varphi}. \tag{4.6}$$

4.2 Surface current and electromagnetic force *(to ToC)*

The surface current is related to the plasma perturbation by

$$\begin{aligned}&\mathbf{i} = \nabla I(\omega,\varphi) \times \mathbf{e}_n = -\frac{1}{a}I'_\omega \mathbf{e}_\varphi + \frac{1}{R}I'_\varphi \mathbf{e}_\omega, \\ &\mu_0 i_{mn,s} = \mu_0\xi_{nm}\left[(m-1)j - \frac{2nB_\varphi}{R}\right]e^{im\omega - in\varphi} = -\frac{im}{a}I_{mn}e^{im\omega - in\varphi}, \\ &I_{mn} = 2\mu_0\xi_{nm}i\left(\frac{m-1}{m}B_\omega - \frac{na}{mR}B_\varphi\right), \quad I(\omega,\varphi) = \Re\left(\sum I_{nm}e^{im\omega - in\varphi}\right).\end{aligned} \tag{4.7}$$

In the real space this relationship can be written in as a first order differential equation

$$2i\mu_0 a\mathbf{B}\nabla\xi + 2\mu_0 B_\omega \xi = I'_\omega. \tag{4.8}$$

The pressure acting on the surface current is

$$p = \mathbf{e}_n \cdot ((\nabla I \times \mathbf{e}_n) \times \mathbf{B}) = (\mathbf{B}\cdot\nabla I) = \frac{B_\omega}{a}\Re\left(\sum (m - nq)I_{mn}e^{im\omega - in\varphi}\right), \tag{4.9}$$

while the sideway force is given by Eq. (2.8). For for the circular plasma cross section, considered here,

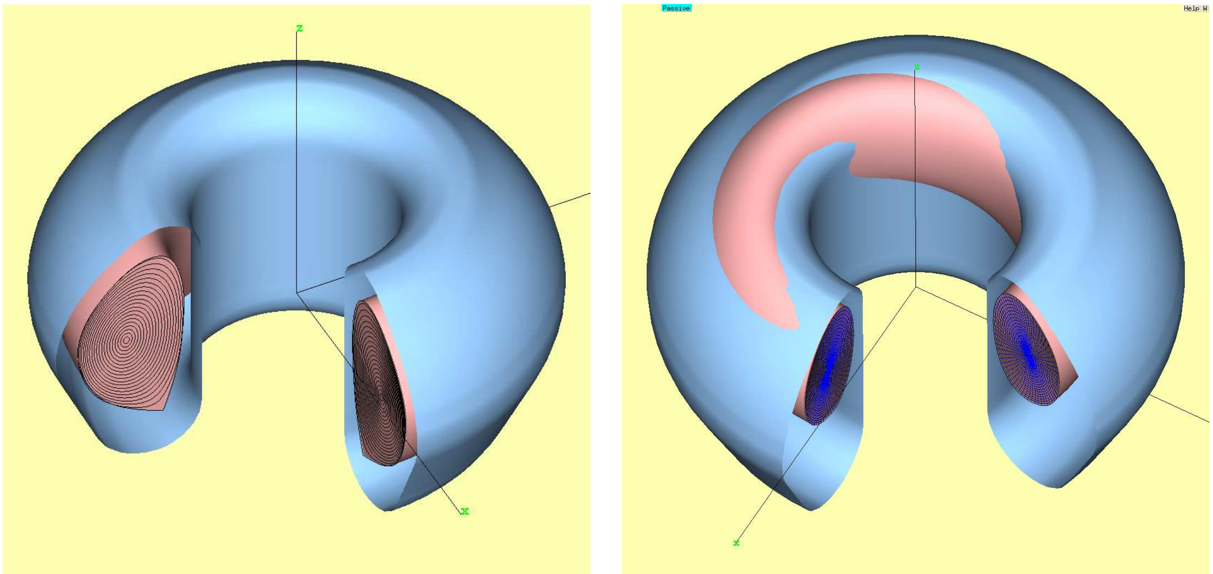
$$p = 2I_{pl}B_\omega \Re\left(\sum \xi_{nm}\frac{nq - m + 1}{m}(m - nq)\,e^{im\omega - in\varphi}\right). \tag{4.10}$$

5 Summary *(to ToC)*

The effects which can be attributed to the existence of the localized forces to the plasma facing components during disruptions were observed on tokamaks sometimes in the form of displacements of the vacuum vessel, asymmetric forces, or broken tiles of plasma facing components. A relatively long lasting $m/n = 1/1$ mode was typically observed in these experiments despite the fact that the q-value ($q \simeq 1.5$) was in the stable range. Such an equilibrium requires sharing of the surface plasma current, excited by the plasma perturbations, with external structures, thus, creating the forces acting on the conducting in-vessel elements.

The presented theory describes a possible mechanism of maintaining a perturbed plasma equilibrium if halo currents are allowed. In particular, the theory gives a relationship, Eq. (4.8), between the plasma displacement and the surface currents. This allows one to calculate the sideway forces acting on the wet spot using Eq. (4.10) and Eqs. (2.8, 3.1).

The calculations of the geometry of the wet spot given the plasma displacement are also important for specifying the topology of the surface current. One example is given in Fig. 5, where the ITER vacuum vessel was simulated by a simple toroidally symmetric shell.



(a) (b)

Figure 5. ITER plasma inside a shell-like model of the plasma facing structure, which fits the inner surface of the vacuum vessel. (a) Equilibrium configuration. (b) The wet zone made by a plasma vertical displacements (due to vertical instability), inward radial displacement (due to loss of plasma current in a given vertical field) displacements, and a kink mode, producing both vertical and sideway forces.

The theory gives a basis for self-consistent calculations of the surface currents, plasma displacement, and the geometry of the wet zone in order to determine stability conditions and numerical values of forces, acting on the wall during disruptions in the real in-vessel environment of tokamaks.

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