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Anisotropic Pressure, Transport, and Shielding of Magnetic Perturbations

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Anisotropic pressure, transport, and shielding of magnetic perturbations

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We compute the effect on a tokamak of applying a nonaxisymmetric magnetic perturbation $\delta\mathbf{B}$. An equilibrium with scalar pressure p yields zero net radial current, and therefore zero torque. Thus, the usual approach, which assumes scalar pressure, is not self-consistent, and masks the close connection which exists between that radial current and the in-surface currents, which provide shielding or amplification of $\delta\mathbf{B}$. Here, we analytically compute the pressure anisoptropy, $p_{||}, p_\perp \neq p$, and from this, both the radial and in-surface currents. The surface-average of the radial current recovers earlier expressions for ripple transport, while the in-surface currents provide an expression for the amount of self-consistent shielding the plasma provides.

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I. INTRODUCTION

As is well known, imposing a nonaxisymmetric magnetic perturbation $\delta\mathbf{B}$ on a tokamak produces a non-ambipolar flux-surface averaged radial current $\langle j^r\rangle \equiv \langle\nabla r\cdot\mathbf{j}\rangle$ due to "ripple transport", expressions for which have been worked out for a range of regimes and mechanisms.[1–7] This radial current produces a toroidal torque $\tau_\zeta \simeq \langle j^r\rangle B_p R/c$ on the plasma, which can change the plasma rotation speed. (Here, R is the major radius, $\mathbf{B}$ is the magnetic field, and $B_p \equiv \hat{\theta}\cdot\mathbf{B}$ is its poloidal component.) Less appreciated is that this same perturbation produces a pressure anisotropy, $p_{||} \neq p_\perp$, (with $p_{||}, p_\perp$ the pressures parallel and perpendicular to the magnetic field $\mathbf{B}$), and that this anisotropy in turn produces both j^r and in-surface currents, which provide a self-consistent contribution to $\delta\mathbf{B}$. Thus, if the phase of the response is correct,this self-consistent response can shield the imposed perturbation. In this paper, we analytically compute expressions for the pressure anisotropy, for the resultant radial and in-surface currents, and for the amount of self-consistent response this implies. Previous work has computed the flux-surface average $\langle j^r\rangle$ of the radial current for a given perturbing field. In the present work, the full flux-surface dependence of both j^r and the in-surface currents is obtained, needed for a calculation of the self-consistent response.

For a fixed, externally-imposed perturbation, it is observed experimentally[8–11] that a rotating plasma can continue to rotate up to some perturbation amplitude, above which the plasma "locks" to the perturbation, which can lead to plasma disruptions. Below this amplitude, the plasma is able to self-consistently shield out the externally-applied field. The self-consistent response computed here provides a criterion for determining this amplitude.

The usual approach in computing the effect of a magnetic perturbation $\delta\mathbf{B}$ on a plasma has been to use an equilibrium with scalar pressure, and to compute the torque from the ripple transport produced by that $\delta\mathbf{B}$. As pointed out in Ref. 12, however, this approach is non-self-consistent, because a scalar-pressure equilibrum produces $j^r = 0$ and correspondingly $\tau_\zeta = 0$. The IPEC code employed in Ref. 12 presently uses a perturbed scalar-pressure equilibrium.[13,14] However, it may be extended to incorporate this self-consistent shielding effect, by including the analytic expressions for $p_{||}, p_\perp$ provided by the present work. The result would be to provide a self-consistent calculation of the plasma response, as here, but with the generality, completeness, and accuracy provided by a numerical tool. The more approximate approach used here has the usual complementary value of analytic calculations, providing explicit formulae from which parametric dependences and additional insights can be obtained.

The remainder of this paper is organized as follows. In Sec. II we introduce the equilibrium equations we need to solve, and the coordinates we use to parametrize the torus. As indicated above, the imposition of $\delta\mathbf{B}(\mathbf{x}, t)$ produces a modification δB of magnetic field strength B, which in turn produces a pressure anisotropy, $p_{||} \neq p_\perp$, computed in Sec. III. This produces a modification $\delta\mathbf{j}$ in the current to satisfy Eq.(1), which via Ampere's law modifies the perturbing magnetic field δB, shielding the plasma, or perhaps, amplifying it. This is studied in Sec. IV. There, it is also demonstrated that $\langle j^r\rangle$ recovers earlier results for "banana-drift" ripple transport.[5–7] A summarizing discussion is given in Sec. V.

II. EQUILIBRIUM AND COORDINATES

We wish to compute the modification of currents and fields in a plasma equilibrium, satisfying force balance for each species s,

$$\mathbf{j}_s \times \mathbf{B} = c\mathbf{f}_s, \tag{1}$$

due to the external application of a magnetic perturbation $\delta\mathbf{B}(\mathbf{x}, t)$, plus the self-consistent response of the plasma. Here, $\mathbf{B}$ is the magnetic field, $\mathbf{j}_s = e_s n_s \mathbf{u}_s$ the current carried by species s, and with $n_s, \mathbf{u}_s$, and e_s the species density, flow velocity, and charge per particle, respectively. $\mathbf{f}_s$ is minus the force per unit volume exerted on species s, by the plasma, the ambipolar electric field, and external forces (see Eq.(2)). $\mathbf{x}$ is the real-space position, parametrized by flux coordinates $\{q^i\} \equiv \{\rho, \theta, \zeta\}$, with θ, ζ the poloidal and toroidal azimuths, resp., and flux-surface label ρ. At times it will be convenient to specialize ρ to $\psi \equiv$ (toroidal flux)$/2\pi$, or to an average geometric minor radius, $r(\rho) \equiv (2\psi/B_0)^{1/2}$, with $B_0 \equiv \bar{B}(r = 0)$ the magnetic field strength on axis. Then $\mathbf{B}$ may be written in the Clebsch representation $\mathbf{B} = \nabla\psi \times \nabla\theta + \nabla\zeta \times \nabla\chi = \nabla\zeta_d \times \nabla\chi$, with $2\pi\chi$ the poloidal flux, Clebsch angle $\zeta_d \equiv \zeta - q\theta$, constant along a field line, $q \equiv \iota^{-1} \equiv d\psi/d\chi$ the safety factor, and ι the rotational transform.

In Ref. 15, the contribution to self-consistent shielding from Coriolus terms ($Mn\mathbf{u}\cdot\nabla\mathbf{u}$) was studied in slab geometry. In the present work, we neglect the Coriolus term in the force-balance, and study the contribution from pressure anisotropy in toroidal geometry. Then $\mathbf{f}$ is given by (species label suppressed)

$$\mathbf{f} = \nabla \cdot \mathbf{P} + en\nabla\Phi + \mathbf{f}_{xt}, \tag{2}$$

with pressure tensor $\mathbf{P} = p\mathbf{1} + \boldsymbol{\pi} = p_\perp \mathbf{1} + (p_{||} - p_\perp)\hat{\mathbf{B}}\hat{\mathbf{B}}$, unit tensor $\mathbf{1}$, viscosity tensor

$\boldsymbol{\pi}$, parallel and perpendicular pressures $p_{||}$ and $p_{\perp}$, scalar pressure $p = (2p_{\perp} + p_{||})/3$, $\hat{\mathbf{B}} \equiv \mathbf{B}/B$, $\Phi(\rho)$ the ambipolar potential, and $\mathbf{f}_{xt}$ an externally-imposed force.

III. CALCULATION OF $p_{||}$ AND $p_{\perp}$

We now compute $p_{||}, p_{\perp}$ due to a magnetic perturbation $\delta B(\mathbf{x}, t)$. One has

$$\begin{bmatrix} n \\ p_{||} \\ p_{\perp} \end{bmatrix} (\mathbf{x}) = \int d\mathbf{v} \begin{bmatrix} 1 \\ M v_{||}^2 \\ M v_{\perp}^2/2 \end{bmatrix} f(\mathbf{z}), \tag{3}$$

with distribution function $f(\mathbf{z})$, $\mathbf{z}(\mathbf{x}, \mathbf{v})$ the (6-D) phase-space position, and velocity $\mathbf{v}$. We first consider collisionless solutions for f, *i.e.*, solutions of the steady-state Vlasov equation. Such solutions are those f_0 which are a function of the constants $\mathbf{I}$ of the motion. For convenience in the following, we specialize ρ to $r(\rho)$, as defined in Sec. II. Then we take $\mathbf{I} = (E, \mu, \bar{r})$, with $E = Mv^2/2 + e\Phi(\mathbf{x})$ the particle energy, $\mu \equiv M v_{\perp}^2/2B$ the magnetic moment, and $\bar{r}$ the drift-orbit averaged value of r. One may write $r = \bar{r} + \delta r(\boldsymbol{\theta}, \mathbf{I})$, with $\boldsymbol{\theta} \equiv (\theta_g, \theta_b, \theta_d)$ the gyrophase, bounce phase, and drift phase, resp.[16] For example, an f_0 having $p_{||} \neq p_{\perp}$, and reducing to a Maxwellian f_M for $T_{||} = T_{\perp}$, is[17]

$$f_0(\mathbf{I}) = \bar{n}(M/2\pi T_{||})^{1/2}(M/2\pi T_{\perp}) \exp[-(E - \bar{\Phi})/T_{||}] \exp[-\mu \bar{B}(T_{\perp}^{-1} - T_{||}^{-1})], \tag{4}$$

where $\bar{n}, T_{||}, T_{\perp}, \bar{\Phi}, \bar{B}$ are functions of $\bar{r}$ alone.

We first neglect δr in $\bar{r}$. Then $f = f_0(E, \mu, r)$, and using

$\int d\mathbf{v} = \frac{2\pi}{M}\int_{-\infty}^{\infty} dv_{||}\int_0^{\infty} d\mu B$, one finds

$$\begin{bmatrix} n \\ p_{||} \\ p_{\perp} \end{bmatrix}(\mathbf{x}) = \begin{bmatrix} 1 \\ T_{||} \\ T_{\perp}/d \end{bmatrix}\bar{n}\exp(-e\Phi_1/T_{||})/d, \tag{5}$$

with $\Phi_1 \equiv \Phi(\mathbf{x}) - \bar{\Phi}$, $d \equiv \tau + (\bar{B}/B)(1-\tau)$ and $\tau \equiv T_{\perp}/T_{||}$. Thus, for $T_{||} = T_{\perp}$ (*i.e.*, $\tau = 1$), one has $d = 1$, and Eq.(5) gives the usual Maxwell-Boltzmann relation $n = \bar{n}\exp(-e\Phi_1/T)$, and $p_{||} = p_{\perp} = p \equiv nT$. (Note that in this case, $[n, p_{||}, p_{\perp}]$ are independent of $B(\mathbf{x})$, as expected by the Bohr-van Leeuwen theorem.) For $T_{||} < T_{\perp}$, higher B on a flux surface reduces $n(\mathbf{x})$, as one expects physically.

Using a similar approach, using an equivalent set $\mathbf{I} \rightarrow (E, \mu, J_b)$, with J_b the bounce action, Hall & McNamara[18] noted that $p_{||,\perp} = p_{||,\perp}(r, \zeta_d, B)$, *i.e.*, that $p_{||}$ and $p_{\perp}$ depend on the distance ℓ along a field line only through the magnitude of $\mathbf{B}$, and they showed that in such cases

$$\partial_B(p_{||}/B) = -p_{\perp}/B^2, \tag{6}$$

a relation satisfied by $p_{||}, p_{\perp}$ in Eq.(5). Multiplying Eq.(6) by $\partial_\ell B$ results in

$$\partial_\ell p_{||} = (p_{||} - p_{\perp})\partial_\ell \ln B,$$

a result proven in Ref. 14 for any anisotropic MHD equilibrium using parallel force balance. Eq.(6) is a stronger result than this, holding for changes in B in any direction, because it makes use of a more specialized set of distribution functions, $f = f_0(\mathbf{I})$.

Higher order contributions

In Ref. 14 it is shown that if $p_{||}$ or $p_{\perp}$ is a function of (r, B) only (hence independent of ζ_d), then that f produces no nonambipolar transport. The local radial currents j^r

may be nonzero, but the complete set of invariants ensures that the flux-surface average $\langle j^r \rangle$ is zero, as for example in an axisymmetric tokamak. Accordingly, we refine f to include two further effects, which do not have this property: (a)finite δr, and (b)finite collisionality ν.

To incorporate both effects, we solve the drift-kinetic equation (dke), as approximately done in Refs. 5–7,16,17. We adopt a standard model for the magnetic field strength in a tokamak with ripple strength δ,

$$B(\mathbf{x}, t) = B_s(r, \theta) + \delta B(\mathbf{x}, t), \tag{7}$$

with symmetric part $B_s \equiv B_0(1 - \epsilon \cos\theta)$, a single-$(n, m)$ harmonic ripple perturbation $\delta B(\mathbf{x}, t) \equiv -B_0 \delta \cos\eta$, $\eta \equiv \eta_{nm} \equiv n\zeta - m\theta - \omega t$, and ω a constant frequency. (For the mode-locking problem, one has $\omega = 0$, but one may envision other, nonzero-ω perturbations being applied, so we retain ω here for greater generality, at little mathematical expense.) For multiple harmonics, this expression for δB may be summed over n, m, and in such cases we explicitly display the subscript nm on η and δ [*e.g.*, see the discussion following Eq.(31)]. The gyro-averaged nonaxisymmetric contribution[3] to the radial motion is

$$\bar{\dot{r}} = (q/M\Omega_g r)\partial_{\zeta_d} \mu B = \hat{\dot{r}} \sin\eta, \tag{8}$$

with gyrofrequency Ω_g and $\hat{\dot{r}} \equiv \delta q n \mu B_0/(M\Omega_g r)$. One obtains this form from the full Fourier expansion $\dot{r}(\boldsymbol{\theta}) = \sum_{\mathbf{l}} v_{\mathbf{l}} \exp(i\mathbf{l}\cdot\boldsymbol{\theta})$ of the radial motion used in the action-angle formalism[16,19] [with $\mathbf{l} \equiv (l_g, l \equiv l_b, l_d)$ the set of gyro, bounce, and drift harmonics], by keeping only the $l_g = 0$ terms. Only the $l_d = n$ terms contribute:

$$\bar{\dot{r}} = \hat{\dot{r}} \sum_{l=-\infty}^{\infty} G_l \sin\eta_l, \tag{9}$$

with $\eta_l \equiv n\zeta_d + l\theta_b - \omega t$, and "orbit-averaging factor"

$$G_l[qN\theta_t(y)] \equiv \oint \frac{d\theta_b}{2\pi} \exp(-il\theta_b) \exp[iqN\theta(\theta_b, y)]. \tag{10}$$

Here, $qN \equiv qn - m$, and $y(r, E, \mu) \equiv (K/\mu B_0 - 1)/2\epsilon + 1/2$ is the well-depth parameter, equal to 0 for deeply-trapped particles, 1 for marginally trapped particles, and > 1 for passing particles. $K \equiv Mv^2/2 = E - e\Phi$ is the kinetic energy.

The linearized dke is

$$(\partial_t + L_{H0})\delta f - C\delta f = -\bar{\dot{r}}\partial_r f_0 + Cf_0, \tag{11}$$

where $L_{H0} \equiv \Omega_d \partial_{\zeta_d} + \dot{\theta}\partial_\theta = \Omega_d \partial_{\zeta_d} + \Omega_b \partial_{\theta_b}$, with $\Omega_{b(d)} \equiv \dot{\theta}_{b(d)}$ the bounce (toroidal drift) frequency. For $T_{||} \neq T_\perp$, f_0 satisfies the usual $L_{H0} f_0 = 0$, but not $Cf_0 = 0$, and so this appears on the right. We henceforth assume $T_{||} - T_\perp$ is small enough to neglect this term. In Refs. 5–7, a Lorentz collision operator C_L was used. For simplicity, here we instead use a Krook operator, $C_K \delta f = -\nu_f \delta f$, with effective collision frequency ν_f. This permits us to approximately treat both the resonant ($D \sim 1/\nu$) and nonresonant ($D \sim \nu$) banana-drift regimes[5,6] at the same time. Then one has $\delta f = \sum_{l=-\infty}^{\infty} f_l \exp i\eta_l + c.c.$, with

$$f_l = -(2i)^{-1} g_0 \hat{\dot{r}} G_l \partial_r f_0 = -\frac{1}{2} r_l \partial_r f_0, \tag{12}$$

with propagator $g_0 \equiv [-i(w - \mathbf{l} \cdot \mathbf{\Omega} + i\nu_{fl})]^{-1}$, $\mathbf{\Omega} \equiv (\Omega_g, \Omega_b, \Omega_d)$, hence $\mathbf{l} \cdot \mathbf{\Omega} = l\Omega_b + n\Omega_d$, $r_l \equiv \hat{\dot{r}} G_l/(\omega - \mathbf{l} \cdot \mathbf{\Omega} + i\nu_{fl})$, and effective collision frequency for the l^{th} bounce harmonic ν_{fl}.

For $\nu_{fl} \gg \omega'_l \equiv \omega - \mathbf{l} \cdot \mathbf{\Omega}$, one has $r_l \simeq \hat{\dot{r}} G_l/(i\nu_{fl})$, hence

$$\delta f = -\hat{\dot{r}}\partial_r f_0 \sum_l (G_l/\nu_{fl}) \sin \eta_l \simeq -(\partial_r f_0/\nu_f)(q/M\Omega_g r)\partial_\zeta(\mu\delta B). \tag{13}$$

From Ref. 6, using C_L, ν_{fl} is approximately $\nu_{fl} \simeq \nu_t[(qN)^2 + (l/2)^2]$, with $\nu_t \equiv \nu/(2\epsilon)$. The final form in (13) is obtained assuming this l-dependence is weak for those l with appreciable G_l, so $\nu_{fl} \simeq \nu_{f0} \equiv \nu_f$.

For $\nu_{fl} \ll \omega_l'$, one has $r_l \simeq \hat{\dot{r}} G_l/\omega_l'$, hence

$$\delta f = -\hat{\dot{r}}\partial_r f_0 \sum_l (G_l/\omega_l') \cos\eta_l \simeq (\partial_r f_0/\omega_0')(nq/M\Omega_g r)\mu\delta B. \tag{14}$$

Analogous to Eq.(13), the final form in (14) is obtained assuming the l-dependence of ω_l' is weak for those l having appreciable G_l. For the present problem, of a mode nearly resonant at a rational surface, one expects $qN\theta_t \lesssim 1$. In that limit, one has $G_l(qN\theta_t) \simeq J_l(qN\theta_t)$ small for $|l| \gtrsim 1$, making the last forms in Eqs.(13) and (14) valid. Under the same assumptions, we obtain from Eq.(12) a single expression for δf, valid for arbitrary ν_f/ω_0':

$$\delta f \simeq (\hat{\dot{r}}/B_0\delta)\partial_r f_0(\tau_\Omega \delta B - \tau_\nu \partial_{n\zeta}\delta B), \tag{15}$$

where $\tau_\Omega \equiv \omega_0'/(\omega_0'^2 + \nu_f^2)$ and $\tau_\nu \equiv \nu_f/(\omega_0'^2 + \nu_f^2)$ have units of time. δf in Eq.(15) will be negligible for passing particles, suppressed by the large jump in Ω_ζ which occurs in going from trapped (trapping state index $\tau = t$) to passing ($\tau = p$).

We now use this expression for δf in Eq.(3). It is convenient to follow Ref. 18 in using $\nu_H \equiv \mu/K$, a constant of motion for $\Phi = 0$, which acts as a pitch-angle variable. Then one has

$$\int d\mathbf{v} = \frac{4\pi}{M^2}(\frac{M}{2})^{1/2} \int_0^\infty dK K^{1/2} \int_{1/B_{tp}}^{1/B} \frac{d\nu_H B}{(1-\nu_H B)^{1/2}}, \tag{16}$$

The lower limit $B_{tp} = B(\theta = \pi)$ in the ν_H-integration is its value at the trapped/passing boundary. For f_0 a local Maxwellian, one has $\partial_r f_0 = -f_0\kappa$, with $\kappa \equiv [\kappa_n + \kappa_T(K/T - 3/2) + \kappa_\phi]$ the radial gradient of f_0, $\kappa_n \equiv -\partial_r \ln n$,

$\kappa_T \equiv -\partial_r \ln T$, and $\kappa_\phi \equiv -(\partial_r e\Phi)/T$. The signs of the κ's are chosen to make their usual value positive.

Inserting Eq.(15) into (3) and using (16), one finds the central result

$$\begin{bmatrix} \delta p_{\|} \\ \delta p_{\perp} \end{bmatrix} (\mathbf{x}) = n_0 T \begin{bmatrix} 2I_{\nu\|} \\ I_{\nu\perp} \end{bmatrix} (I_K/\pi^{1/2})(-\tau_\Omega \delta B + \tau_\nu \partial_{n\zeta} \delta B)/(B\delta)\hat{\dot{r}}_T, \tag{17}$$

where $\hat{\dot{r}}_T \equiv \hat{\dot{r}}(\mu B_0 \to T) = \delta qnT/(M\Omega_g r)$, $I_{\nu\|} \equiv \int_{B/B_{tp}}^1 du(1-u)^{1/2}u \simeq (2/3)(1-B/B_{tp})^{3/2}$, $I_{\nu\perp} \equiv \int_{B/B_{tp}}^1 du(1-u)^{-1/2}u^2 \simeq 2(1-B/B_{tp})^{1/2}$, and $I_K \equiv \int_0^\infty dx x^{5/2} e^{-x}[\kappa_n + \kappa_T(x-3/2) + \kappa_\phi] = \Gamma(7/2)\bar{\kappa}$, with $\bar{\kappa} \equiv [\kappa_n + \kappa_T(7/2-3/2) + \kappa_\phi]$, and $\Gamma(7/2) = (15/8)\pi^{1/2}$ the Gamma function with argument (7/2). The time constants $\tau_{\Omega,\nu}$ are defined at Eq.(15). For simplicity we have neglected the K and ν_H-dependences of these. From this, one notes that $\delta p_{\|}/\delta p_\perp = 2I_{\nu\|}/I_{\nu\perp} = 2(1-B/B_{tp})/3$, where for the model tokamak field (7), one has $(1-B/B_{tp}) = \epsilon(1+\cos\theta)//(1+\epsilon) \simeq 2\epsilon\cos^2(\theta/2)$, and thus $\delta p_{\|}/\delta p_\perp \ll 1$. The perturbed pressure $\delta p_\perp$ is roughly the unperturbed pressure $n_0 T$, times a factor $I_{\nu\perp}$ of order the fraction $F_t \sim \epsilon^{1/2}$ of trapped particles, times a thermal force factor I_K, times the distance $\hat{\dot{r}}_T\ \tau_{\Omega,\nu}$ a particle can drift radially in a time $\tau_{\Omega,\nu}$. From this result we will compute the perturbed current, radial fluxes, and magnetic field.

IV. PERTURBED FLOWS AND SHIELDING

We now use Eq.(17) in Eqs.(1) and (2) to compute the perturbed flows and currents, as outlined in Sec. II. One has

$$\nabla \cdot \mathbf{P} = \nabla p + \nabla \delta p_\perp + (p_{\|} - p_\perp)\boldsymbol{\kappa} + \hat{\mathbf{B}}\mathbf{B} \cdot \nabla((p_{\|} - p_\perp)/B), \tag{18}$$

where $\delta p_\perp \equiv p_\perp - p$ and $\boldsymbol{\kappa} \equiv \hat{\mathbf{B}} \cdot \nabla\hat{\mathbf{B}}$ is the field-line curvature. Dropping the external force $\mathbf{f}_{xt}$, and taking $B^{-1}\hat{\mathbf{B}}\times$Eq.(1), the first 3 terms on the right side of (18) give nonzero contributions:

$$\mathbf{j}_\perp = B^{-1}\hat{\mathbf{B}} \times \mathbf{f} \equiv \mathbf{j}_{\perp p} + \mathbf{j}_{\perp A} + \mathbf{j}_{\perp B}, \tag{19}$$

where $\mathbf{j}_{\perp p} \equiv (c/B)\hat{\mathbf{B}} \times (\nabla p + en\nabla\Phi)$ is the usual equilibrium perpendicular current for a scalar pressure, and $\mathbf{j}_{\perp A} \equiv (c/B)\hat{\mathbf{B}} \times \nabla\delta p_\perp$ and $\mathbf{j}_{\perp B} \equiv (c/B)\hat{\mathbf{B}} \times \boldsymbol{\kappa}\delta p_{\perp||}$ modify this for nonisotropic pressure. Here, $\delta p_{\perp||} \equiv (p_\perp - p_{||})$, approximately equal to $\delta p_\perp$, in view of our findings at (17).

We specialize to Boozer flux coordinates $\{q^i\} \equiv \{\rho, \theta, \zeta\}$. We write $\mathbf{B}$ in its co- and contravariant forms[20]

$$\begin{aligned}\mathbf{B} &= I\nabla\theta + G\nabla\zeta + \psi'\beta_*\nabla\rho \equiv I\mathbf{e}^\theta + G\mathbf{e}^\zeta + \psi'\beta_*\mathbf{e}^\rho \\ &= \psi'\nabla\rho \times \nabla\theta + \chi'\nabla\zeta \times \nabla\rho \equiv \mathcal{J}^{-1}(\psi'\mathbf{e}_\zeta + \chi'\mathbf{e}_\theta),\end{aligned} \tag{20}$$

with $\psi' \equiv \partial_\rho\psi$, $\chi' \equiv \partial_\rho\chi$, covariant basis vectors $\mathbf{e}^i \equiv \nabla q^i$, the reciprocal contravariant set $\mathbf{e}_i \equiv \mathcal{J}\mathbf{e}^j \times \mathbf{e}^k$, with $\mathcal{J} \equiv (\mathbf{e}^\rho \cdot \mathbf{e}^\theta \times \mathbf{e}^\zeta)^{-1}$ the Jacobian $|d\mathbf{x}/d\mathbf{q}|$. and the indices i, j, k run cyclically in the definition of $\mathbf{e}_i$. For Boozer coordinates, $\mathcal{J} = (G\psi' + I\chi')/B^2$. Then using the covariant form for $\mathbf{B}$ in (20), one finds

$$\mathbf{j}_{\perp p} = \frac{F}{B^2}(G\nabla\zeta \times \nabla\rho - I\nabla\rho \times \nabla\theta), \tag{21}$$

where $F(\rho) \equiv c(p' + en_0\Phi')$. Dotting this with $\mathbf{e}^i$, one reads off the components $j^i_{\perp p}$, $j^\theta_{\perp p} = \mathcal{J}^{-1}(F/B^2)G,\ j^\zeta_{\perp p} = -\mathcal{J}^{-1}(F/B^2)I$, and $j^\rho_{\perp p} = 0$. The last of these confirms the statement that scalar pressure yields 0 radial current.

We compute the parallel portions of currents $\mathbf{j}_l$, (with $l \to \{p, A, B\}$), by requiring each portion to satisfy $\nabla \cdot \mathbf{j}_l = 0$. As usual, this may be written as a magnetic differential equation for $g_l \equiv j_{||l}/B$,

$$\mathbf{B} \cdot \nabla g_l = -\nabla \cdot \mathbf{j}_{\perp l}. \tag{22}$$

Using the general expressions $\mathbf{B} \cdot \nabla g = \mathcal{J}^{-1}(\chi' \partial_\theta + \psi' \partial_\zeta) g$ and $\nabla \cdot \mathbf{j} = \mathcal{J}^{-1} \partial_i \mathcal{J} j^i$ (summation over repeat indices implied), one has from Eqs.(21), (22)

$$(\chi' \partial_\theta + \psi' \partial_\zeta) g_p = -F(G \partial_\theta B^{-2} - I \partial_\zeta B^{-2}). \tag{23}$$

Following Ref. 20, we Fourier decompose B^{-2},

$$B^{-2} = B_0^{-2}[1 + \Sigma' \delta_{nm} \cos \eta_{nm}], \tag{24}$$

with $\eta_{nm} \equiv n\zeta - m\theta + \lambda_{nm}$, and Σ' denotes a sum over all n and $m \geq 0$ excluding $(n, m) = (0, 0)$. Then using this in Eq.(23), one finds

$$g_p \equiv j_{||p}/B = g_{p0} + \frac{F}{B_0^2 \psi'} \Sigma' \frac{In + Gm}{n - \iota m} \delta_{nm} \cos \eta_{nm}, \tag{25}$$

slightly generalizing the result in Ref. 20 to one valid for each species. Here, $g_{p0}(\rho)$ is the $(0, 0)$-component of g_p, as yet undetermined.

The procedure is similar, but slightly more involved, for $\mathbf{j}_{A,B}$, the difference being that these have nonzero radial components $j^\rho \equiv \mathbf{e}^\rho \cdot \mathbf{j}$. The flux-surface average of these give the nonambipolar particle fluxes. Analogous to Eq.(21), one finds

$$\begin{aligned} \mathbf{j}_{\perp A} \; = \; & \frac{c \delta p'_\perp}{B^2}(G \nabla \zeta \times \nabla \rho - I \nabla \rho \times \nabla \theta) \\ & + \frac{c}{B^2}[(I \partial_\zeta - G \partial_\theta) \delta p_\perp \nabla \theta \times \nabla \zeta + \beta_* \nabla \psi \times (\nabla \theta \partial_\theta + \nabla \zeta \partial_\zeta) \delta p_\perp], \end{aligned} \tag{26}$$

where $\delta p'_\perp \equiv \partial_\rho \delta p_\perp$. The final term in (26), proportional to β_*, lies in a flux surface, as does the dominant term on the first line, and so will be neglected. From this we then read off $j^{\theta}_{\perp A} \simeq \mathcal{J}^{-1}(c\delta p'_{\perp}/B^2)G$, $j^{\zeta}_{\perp A} \simeq -\mathcal{J}^{-1}(c\delta p'_{\perp}/B^2)I$, and $j^{\rho}_{\perp A} = \mathcal{J}^{-1}(B^{-2})(I\partial_{\zeta} - G\partial_{\theta})c\delta p_{\perp}$.

Similarly, using that $\boldsymbol{\kappa} \simeq \nabla_{\perp}B/B$, and again neglecting the term in β_*, for $\mathbf{j}_B$ one has

$$\begin{aligned} \mathbf{j}_{\perp B} \;\simeq\; & \frac{c\delta p_{\perp||}}{2}(G\nabla\zeta \times \nabla\rho - I\nabla\rho \times \nabla\theta)\partial_{\rho}(B^{-2}) \\ & +[(I\partial_{\zeta} - G\partial_{\theta})(B^{-2})\nabla\theta \times \nabla\zeta, \end{aligned} \tag{27}$$

from which one reads off $j^{\theta}_{\perp B} \simeq \mathcal{J}^{-1}(c\delta p_{\perp||}/2)G\partial_{\rho}(B^{-2})$, $j^{\zeta}_{\perp B} \simeq -\mathcal{J}^{-1}(c\delta p_{\perp||}/2)I\partial_{\rho}(B^{-2})$, and $j^{\rho}_{\perp B} \simeq \mathcal{J}^{-1}(c\delta p_{\perp||}/2)(I\partial_{\zeta} - G\partial_{\theta})(B^{-2})$.

Analogous to Eq.(23), one then finds, noting a cancellation of the 2nd-order derivatives on $\delta p_{\perp}$ or B^{-2},

$$\begin{aligned} (\chi'\partial_{\theta} + \psi'\partial_{\zeta})g_A \;=\; & c\{-\delta p'_{\perp}(G\partial_{\theta} - I\partial_{\zeta})(B^{-2}) \\ & -(\partial_{\zeta}\delta p_{\perp})\partial_{\rho}(I/B^2) + (\partial_{\theta}\delta p_{\perp})\partial_{\rho}(G/B^2)\} \end{aligned} \tag{28}$$

for $\mathbf{j}_A$, and the similar expression

$$\begin{aligned} (\chi'\partial_{\theta} + \psi'\partial_{\zeta})g_B \;=\; & \frac{c}{2}\{-\partial_{\rho}(B^{-2})(G\partial_{\theta} - I\partial_{\zeta})\delta p_{\perp||} \\ & -(\partial_{\zeta}(B^{-2})\partial_{\rho}(I\delta p_{\perp||}) + (\partial_{\theta}(B^{-2})\partial_{\rho}(G\delta p_{\perp||})\} \end{aligned} \tag{29}$$

for $\mathbf{j}_B$. The second lines of Eqs.(28) and (29) are from the additional contributions from $j^{\rho}_{A,B}$. Using the approximation $\delta p_{\perp||} \simeq \delta p_{\perp}$ in Eq.(29), Eqs.(28) and (29) may be combined, yielding

$$\begin{aligned} (\chi'\partial_{\theta} + \psi'\partial_{\zeta})g_{AB} \;=\; & c\{B^{-2}(G'\partial_{\theta} - I'\partial_{\zeta})\delta p_{\perp} + \frac{1}{2}\delta p_{\perp}(G'\partial_{\theta} - I'\partial_{\zeta})B^{-2} \\ & -\frac{1}{2}\delta p'_{\perp}(G\partial_{\theta} - I\partial_{\zeta})B^{-2} + \frac{1}{2}(\partial_{\rho}B^{-2})(G\partial_{\theta} - I\partial_{\zeta})\delta p_{\perp}\}, \end{aligned} \tag{30}$$

where $g_{AB} \equiv g_A + g_B$.

We consider the (θ, ζ) dependence of (30). We write $\delta p_\perp$ in Eq.(17) as

$$\delta p_\perp = -p_\Omega(\delta B/B) + p_\nu(\partial_{n\zeta}\delta B/B), \tag{31}$$

with the main (θ, ζ) dependence held in the δB factors, and coefficients $p_{\Omega,\nu} \propto \tau_{\Omega,\nu}$ given by Eq.(17). While from Eq.(7) or (24) the factor B^{-2} also has a contribution from the δB variation ($n \neq 0$), the dominant contributions come from the B_s term. Keeping only these, one has from Eq.(7), $B^{-2} \simeq B_0^{-2}(1 + 2\epsilon(\rho)\cos\theta) + h.o.$ Regarding δB, an applied perturbation with a single $\eta \equiv \eta_{nm}$ component, $\delta\mathbf{B} \simeq \tilde{B}_r \sin\eta\nabla r$, corresponds, via Ampere's law, to a current $\delta\mathbf{j} = (c/4\pi)\nabla \times \delta\mathbf{B} \simeq (c/4\pi)\tilde{B}_r\cos\eta\nabla\eta \times \nabla r$, and produces a surface deformation $\xi_r \simeq -\tilde{\xi}_r\cos\eta$ in the ∇r direction, with nonresonant (*i.e.*, far from $\iota(r) = \iota_{nm} \equiv n/m$) amplitude $\tilde{\xi}_r^{nres} = R\tilde{b}_r/(n - m\iota)$ (with $\tilde{b}_r \equiv \tilde{B}_r/B_0$), and resonant amplitude $\tilde{\xi}_r^{res} = \pm(2\tilde{b}_r R q_0^2/m q_0')^{1/2}$. This deformation in turn changes $B \equiv |\mathbf{B}|$ on the surface by $\delta B^{(b)} = B_s(r + \xi_r, \theta) - B_s(r, \theta) \simeq (\tilde{\xi}_r/R)B_0\cos\eta_{nm}\cos\theta$. Using this in Eq.(8) gives $\bar{\dot{r}} = \bar{\dot{r}}^{(b)} = \hat{\dot{r}}\sin\eta_{nm}\cos\theta = \hat{\dot{r}}\frac{1}{2}\sum_{m'=m\pm1}\sin\eta_{nm'}$, with $\delta^{(b)} = (-\tilde{\xi}_r/R)$ in $\hat{\dot{r}} = \hat{\dot{r}}^{(b)}$. This again yields Eqs.(9) and (10), but with $G_l(qN_{nm}\theta_t)$ there replaced with $G_l^{(b)}(qN_{nm}\theta_t) = \frac{1}{2}\sum_{m'=m\pm1}G_l(qN_{nm'}\theta_t)$. Near a rational surface $\iota = \iota_{nm}$, such as studied in Ref. 7, η_{nm} is approximately independent of $\theta(\theta_b)$, $\eta_{nm} \simeq n\zeta_d - \omega t$, hence the bounce-average $\langle\bar{\dot{r}}\rangle_b$ of $\bar{\dot{r}}$ is $\langle\bar{\dot{r}}\rangle_b = \hat{\dot{r}}^{(b)}G_0^{(b)}\sin\eta_{l=0} \simeq \hat{\dot{r}}^{(b)}\langle\cos\theta\rangle_b\sin\eta_{nm}$. Here, $\langle\cos\theta\rangle_b = 2E(y^{1/2})/K(y^{1/2}) - 1$ is the bounce-average of $\cos\theta$,[21] and K and E the complete elliptic integrals. This is close to the form given in Ref. 7 [at Eq.(10)], here denoted by superscript (c), but with a slightly different magnetic field model. Converting that expression from its radial coordinate Ψ to r, and multiplying the integral $\langle..\rangle$ used there by[6] $\Omega_b/2\pi = (4qRK(y^{1/2})^{-1}(\mu B_0\epsilon/M)^{1/2}$ to convert to the bounce-average used here, that expression is $\langle \bar{\dot{r}} \rangle_b = 4(I/\Omega_g)(dr/d\chi)(\mu B_0/M\Delta)^{1/2}(2E - K)(\partial_{\alpha_d}\Delta)\Omega_b/2\pi \simeq \hat{\dot{r}}^{(c)}\langle \cos\theta \rangle_b \sin\eta_{nm}$, with $\alpha_d \equiv \theta - \zeta/q_{nm} = -\eta_{nm}/m$, $\Delta \equiv \epsilon + \delta^{(b)}(\bar{\Psi} + \cos\eta_{nm})^{1/2}$, and with $\delta^{(c)} = (-\tilde{\xi}_r^{res}/2R)(\bar{\Psi} + \cos\eta_{nm})^{-1/2}$ in $\hat{\dot{r}} = \hat{\dot{r}}^{(c)}$.

To consider shielding, we are especially interested in the components of $g_{AB} = j_{||AB}/B$ at the same (n,m) harmonic as the applied current $j_{||} \sim \cos\eta$. Since $\delta B^{(b)}$ has an additional factor $\cos\theta$, it has contributions only at $m \pm 1$ sidebands of this. Only the extra $\cos\theta$ harmonic from B^{-2}, occurring in the first and fourth terms in Eq.(30), can then shift the overall (θ,ζ) dependence back to the η_{nm} fundamental. In these terms, one has a dependence $\partial_\eta g_{AB} \sim \cos\theta\partial_\eta(-p_\Omega \cos\eta\cos\theta + p_\nu\partial_\eta \cos\eta\cos\theta) = (p_\Omega \sin\eta - p_\nu \cos\eta)(1 + \cos 2\theta)/2$. Integrating with respect to η, one sees that the p_Ω term yields an in-phase contribution $\sim \cos\eta$ to g_{AB}, and the p_ν term yields a $\pi/2$ out-of-phase contribution $\sim \sin\eta$. Denoting by g_{AB}^{nm} this “diagonal” contribution, one finds

$$g_{AB}^{nm} = \frac{c}{2B_0^2\psi'}\frac{[2\epsilon(G'm + I'n) + \epsilon'(Gm + In)]}{n - \iota m}\frac{\tilde{\xi}_r}{R}(p_\Omega \cos\eta_{nm} + p_\nu \sin\eta_{nm}). \quad (32)$$

The electrostatic susceptibility (reinstating species label) $K_s \equiv -\delta\phi_s/\delta\phi$ gives the potential $\delta\phi_s = (4\pi/k^2)\delta\rho_s$ produced by the self-consistent response to $\delta\phi$ of species s, giving the shielding equation $\delta\phi_{xt} = \delta\phi - \sum_s \delta\phi_s = (1 + \sum_s K_s)\delta\phi$. In analogy, one may define a current response function $K_{js} \equiv -j_{||ABs}^{nm}/\delta j_{||}$, so that

$$\delta j_{||xt} = \delta j_{||} - \sum_s \delta j_{||ABs}^{nm} = (1 + \sum_s K_{js})\delta j_{||}. \quad (33)$$

Defining $k_\theta \equiv m/r$, a complex pressure $p_c \equiv (p_\Omega - ip_\nu)$, and using (32), one finds

$$K_{js} = -\frac{2\pi p_{cs}}{B_0^2}\frac{[2\epsilon(G'm + I'n) + \epsilon'(Gm + In)]}{\psi'(n - \iota m)}\frac{\tilde{\xi}_r}{\tilde{b}_r k_\theta R}. \quad (34)$$

For $|K_j| \sim 1$, the current produced by the ripple field becomes comparable to that needed to produce the ripple. so shielding can occur. Total shielding occurs for $K_j =$

-1. One has $In/Gm \simeq (rB_p/RB_t)(n/m) \simeq \epsilon^2 \iota\iota_{nm} \ll 1$, hence one may neglect the terms in I. And for a low-beta equilibrium, one has $\epsilon G'/(\epsilon' G) \ll 1$. Using in addition the nonresonant expression $\tilde{\xi}_r^{nres}$, Eq.(34) simplifies to

$$K_{js} \simeq -\frac{2\pi p_{cs}}{B_0^2} \frac{1}{(n - \iota m)^2}. \tag{35}$$

The resonant denominators appearing in Eq.(35) come from integration along an unperturbed field line to obtain nonresonant expressions for $\tilde{\xi}_r$ and $j_{||AB}^{nm}$. For present purposes, we heuristically extend this expression from validity only far from resonance to one valid near resonance too, by replacing the factor $(n - \iota m)^{-2}$ in Eq.(35) with a resonance factor $\mathcal{R}^2$, with $\mathcal{R}^2(\rho, \tilde{b}_r) = (\min[|\tilde{\xi}_r^{nres}|, |\tilde{\xi}_r^{res}|]/(R\tilde{b}_r))^2 = \min[(n - \iota m)^{-2}, (2q_0^2/\tilde{b}_r Rmq_0')]$. Making this replacement in (35), considered as a function of ρ, $|K_j|$ will exhibit peaks at resonant surfaces, whose magnitude ($\propto 1/\tilde{b}_r$) decreases as $\tilde{b}_r$ increases. For a given β or pressure, if the maxima of $|K_j|(\rho)$ fall below 1, the plasma is incapable of fully shielding out a perturbation above some maximum amplitude $\tilde{b}_r$. The larger the pressure, the larger p_c, so the greater the capacity of the plasma to produce shielding currents, consistent with experimental observations.

Radial fluxes

From Eqs.(21), (26), and (27), the radial current produced by the perturbation for each species is given by

$$j^\rho = j_p^\rho + j_A^\rho + j_B^\rho = \mathcal{J}^{-1} c[(B^{-2})(I\partial_\zeta - G\partial_\theta)\delta p_\perp + (\delta p_\perp/2)(I\partial_\zeta - G\partial_\theta)(B^{-2})]. \tag{36}$$

We take the flux-surface average of this to obtain the net radial current $\langle j^\rho \rangle = e\langle \Gamma \rangle$,

$$\langle j^\rho \rangle \equiv V'^{-1} \oint d\theta d\zeta \mathcal{J} j^\rho = V'^{-1} c \oint d\theta d\zeta [(B^{-2})\partial_y \delta p_\perp + (\delta p_\perp/2)\partial_y (B^{-2})],$$

$$= -V'^{-1}c \oint d\theta d\zeta (\delta p_\perp/2)\partial_y (B^{-2}), \tag{37}$$

where $V' \equiv \oint d\theta d\zeta \mathcal{J} \simeq (2\pi)^2 rR$, $\partial_y \equiv (I\partial_\zeta - G\partial_\theta)$, and in going from the first to the second line we have integrated the first term (from j^ρ_A) by parts with respect to θ and ζ.

From (7), $B^{-2} \simeq B_0^{-2}(1 - 2\delta B_s/B_0 - 2\delta B/B_0)$, with $\delta B_s \equiv B_s - B_0 = -B_0\epsilon\cos\theta$. The average selects the (0,0) component of the integrand, and thus comes from those Fourier components of B^{-2} with the same δB-dependence as the $\delta p_\perp$ factor. The $\delta B_s/B_0$ term thus makes 0 contribution to the average. Thus,

$$\langle j^\rho \rangle \simeq \frac{c}{V' B_0^3} \oint d\theta d\zeta \delta p_\perp \partial_y \delta B = \frac{cp_\nu}{V' B_0^4} \oint d\theta d\zeta (\partial_{n\zeta}\delta B)(\partial_y \delta B), \tag{38}$$

where we note that the p_Ω-term in Eq.(31) makes 0 contribution, since it is out of phase. For a single (n, m) harmonic, one has $\partial_y \delta B/B_0^2 = [(nI + mG)/B_0](\partial_\eta \delta B/B_0) \simeq nRq_{nm}\partial_\eta \delta B/B_0$, with $q_{nm} \equiv m/n$. Reading off p_ν from Eq.(17), we find

$$\langle j^\rho \rangle = e\langle \Gamma \rangle \simeq e\frac{q_{nm}}{q}\bar{D}_{bd}\bar{\kappa} n_0, \tag{39}$$

where $\bar{D}_{bd} \equiv \oint \frac{d\theta d\zeta}{(2\pi)^2} I_{\nu\perp}\bar{\dot{r}}_T^2 \tau_\nu \Gamma(7/2)/\pi^{1/2} \sim \epsilon^{1/2}\hat{\dot{r}}_T^2\tau_\nu$ is the banana-drift diffusion coefficient[5,6], valid in both the $1/\nu$ (resonant) and ν^1 (nonresonant) regimes, with $\bar{\dot{r}}_T \equiv (\hat{\dot{r}}_T/\delta)(\partial_\eta \delta B/B_0)$ the thermal value of the radial drift velocity $\bar{\dot{r}}$. The expression for the flux in Ref. 7 [at Eq.(12)] is this same banana-drift flux, specialized to the $1/\nu$ regime, and with ripple strength $\delta \to \delta^{(c)}$ in $\hat{\dot{r}}$, and (θ, ζ)-dependence $\delta B \to \delta B^{(c)} \sim \cos\eta\cos\theta$ produced near a rational surface, as discussed above, instead of the $\delta B \sim \cos\eta$ dependence assumed in Refs. 5,6.

V. SUMMARY

In this paper, we have developed analytic expressions (Eqs.(17)) for the parallel and perpendicular pressures $p_{||}, p_\perp$ due to a magnetic perturbation $\delta\mathbf{B}$ imposed on an axisymmetric tokamak field. As discussed, a scalar pressure produces zero net non-ambipolar radial current $\langle j^r \rangle$, as do certain classes of anisotropic pressures, so these expressions for $p_{||}, p_\perp$ are necessary to provide a self-consistent picture of the plasma transport and equilibrium changes which are produced under the application of an external magnetic perturbation. Using the tensor-pressure equilibrium equations, from $p_{||}, p_\perp$ we have computed the radial and in-surface currents produced by the corresponding pressure anisotropy. The expressions are valid in both the $1/\nu$ (resonant) and ν^1 (nonresonant) banana-drift regimes. The radial current j^r has a portion ($\propto p_\Omega$) which gives 0 contribution to $\langle j^r \rangle$, and so to the net toroidal torque. We have shown that the second portion ($\propto p_\nu$) produces a $\langle j^r \rangle$ which recovers earlier expressions for banana-drift transport fluxes,[5–7]. From the in-surface currents, we have obtained an approximate analytic expression for the current response function K_j (Eq.(34)), which measures the size and phase of the self-consistent response. The calculation leaves out the effect of "off-diagonal" contributions to the plasma response, which would require either a more involved analytic treatment, or a more complete numerical calculation, to provide. Heuristically extending K_j to be valid near as well as far from a rational surface, the expression is consistent with the experimental observation that higher-pressure plasmas can shield out applied perturbations up to larger amplitudes. The expressions for $p_{||}, p_\perp$ may be used analytically, as here, or incorporated into a perturbed equilibrium code such as IPEC to obtain self-consistent expressions for the currents and shielding effects.

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