Princeton Plasma Physics Laboratory

PPPL- 4648

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A Magnetic Diagnostic Code for 3D Fusion Equilibria

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July, 2011

Prepared for the U.S. Department of Energy under Contract DE-AC02-09CH11466.

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Abstract

A synthetic magnetic diagnostics code for fusion equilibria is presented. This code calculates the response of various magnetic diagnostics to the equilibria produced by the VMEC and PIES codes. This allows for treatment of equilibria with both good nested flux surfaces and those with stochastic regions. DIAGNO v2.0 builds upon previous codes through the implementation of a virtual casing principle. The codes is validated against a vacuum shot on the Large Helical Device where the vertical field was ramped. As an exercise of the code, the diagnostic response for various equilibria are calculated on the Large Helical Device (LHD).

Keywords: fusion, diagnostics, simulation, equilibrium

1. Introduction

The calculation of synthetic magnetic diagnostics for three dimensional magnetic fields in fusion devices is important for both stellarator and tokamaks with non-axisymmetric field coils. Simulation of magnetic signals for two-dimensional toroidal configurations is a well treated problem in magnetically confined fusion. This has allowed for the development of codes which fit magnetic equilibria to various plasma diagnostics [1, 2]. Three dimensional fields require significantly larger computational efforts to achieve sim-

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ilar goals. Thus only devices which had inherently three-dimensional fields (stellarators and heliotrons) required these more computationally expensive codes. It has now been recognized that otherwise axisymmetric configurations can benefit from three-dimensional fields [3, 4] motivating the development of computational tools capable of handling such configurations. Recent work on edge localized modes suggest that 3D physics plays an important role in future fusion devices such as ITER. The code presented in this paper, DIAGNO v2.0, interfaces with both the VMEC [5] and PIES [6] codes to calculate the magnetic diagnostic response to their three dimensional equilibria. This allows these 3D equilibrium codes to be used for reconstruction purposes. This code also allows for the examination of magnetic diagnostic the response to islands and stochastic regions. Previous diagnostic codes were only coupled to equilibrium codes which did not posses such featrues (VMEC) or lacked the ability to simulate the total diagnostic response (EXTENDER) [6].

The development of this code was necessitated by limitations present in the original DIAGNO code [7] and the development of more generalized methods for treating problem of calculating the plasma response from a given equilibrium. The DIAGNO code calculated the plasma response of the magnetic field (external to the plasma) using three pieces of information (from VMEC): a potential on the equilibrium surface, a current placed on the magnetic axis, and the vacuum field on the surface (via Biot-Savart's law). The scalar potential placed on the plasma boundary limited the applicability of the method to the VMEC code. The use of Biot-Savart's law made the calculation grow as the size of the field coil detail. Recently a code was developed which utilized a virtual casing principle [8] allowing for the calculation of the external magnetic field with only the specification of the plasma boundary and the magnetic field on that boundary (EXTENDER). As DIAGNO could already handle the details of calculating various diagnostic responses, the decision to modify the DIAGNO code to calculate external fields using a virtual casing principle was made.

This paper discusses the DIAGNO v2.0 code along with providing a benchmark against a vacuum shot on the Large Helical Device (LHD). The LHD is a ten field period helical fusion device with superconducting coils which has been in operation in Japan since 1998 [9]. Section 2 discusses the methodology of the code. Details of it's calculation of the field on the equilibrium boundary are provided for both VMEC and PIES equilibria. Section 3 compares the calculations against a vacuum shot on LHD. Section 4 summarizes the results.

2. Method

The DIAGNO v2.0 code employs a virtual casing principle to calculate the fields external to a given plasma equilibrium. This allows the calculation of the magnetic plasma response with only the knowledge of the total magnetic field on the boundary, providing a very simple interface to many existing codes. Additionally, the code can also do a direct Biot-Savart calculation of the diagnostic response due to field coil energization. This allows for a calculation of a mutual inductance matrix for all diagnostics. The Biot-Savart calculation is performed using a method utilized in many other fusion codes [10]. This method utilizes a compact expression for the Biot-Savart magnetic field and vector potential which is singular only on the line segment. The code is currently designed to calculate the diagnostic response of magnetic field probes, flux loops, diamagnetic loops, segmented Rogowski coils, and Mirnov arrays.

At the numerical boundary between the plasma and vacuum the magnetic field must obey

$$\hat{n} \times \left(\vec{B}_{out} - \vec{B}_{in}\right) = \mu_0 \vec{K}_{surf}, \tag{1}$$

where $\hat{n}$ is the surface normal vector pointing from the boundary (in) to the vacuum region (out). If this surface is a flux surface then the solenoidal constraint is identically satisfied ($\nabla \cdot \vec{B} = 0$). This is true of codes such as VMEC. For codes in which the boundary is not a flux surface, such as PIES, a dipole moment density must also be included. This scalar quantity can be calculated from

$$\left(\vec{B}_{out} - \vec{B}_{in}\right) \cdot \hat{n} = \mu_0 \sigma_{dipole}. \tag{2}$$

As we are concerned with only the plasma response, the external field may be assumed to be zero ($B_{out} = 0$). This is the superconducting shell argument. The source terms for the external field (due to the plasma) become

$$\vec{K}_{surf} = \frac{1}{\mu_0} \vec{B}_{surf} \times \hat{n} \tag{3}$$

and

$$\sigma_{dipole} = -\frac{1}{\mu_0} \hat{n} \cdot \vec{B}_{surf}. \tag{4}$$

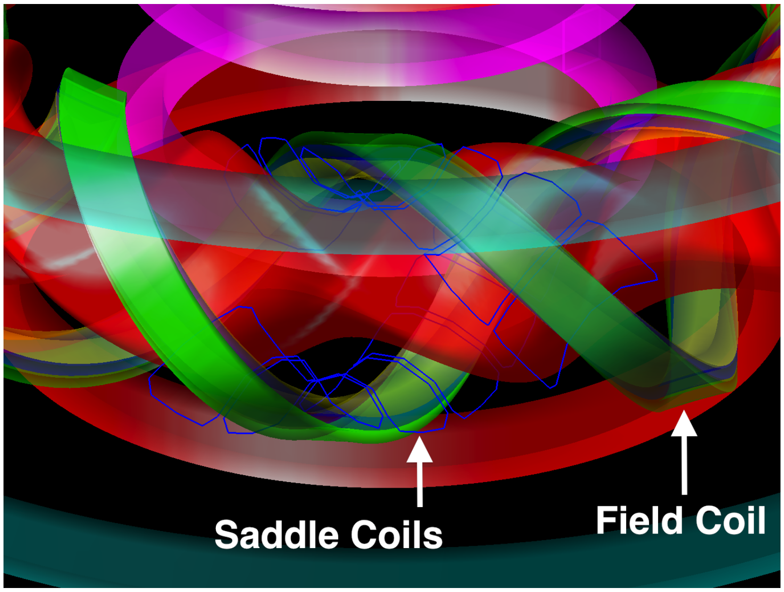

Figure 1: Saddle loops on the Large Helical Device. One of two saddle loop arrays on the LHD (blue lines). A sample equilibrium plasma surface is depicted in red.

The surface current ($\vec{K}$) and dipole density (σ_{dipole}) are then integrated over the plasma surface to find either the magnetic field

$$\vec{B}\left(\vec{x}\right) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}' \times \left(\vec{x}-\vec{x}'\right)}{\left|\vec{x}-\vec{x}'\right|^3} dA' + \frac{\mu_0}{4\pi}\int \frac{\sigma'_{dipole}\left(\vec{x}-\vec{x}'\right)}{\left|\vec{x}-\vec{x}'\right|^3} dA' \tag{5}$$

or vector potential

$$\vec{A}\left(\vec{x}\right) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}'}{\left|\vec{x}-\vec{x}'\right|} dA' + \frac{\mu_0}{4\pi}\int \frac{\sigma'_{dipole}}{\left|\vec{x}-\vec{x}'\right|} \hat{n} dA'. \tag{6}$$

In both equations the prime ($'$) denotes quantities on the boundary surface and integration is carried out over the entire surface.

The DIAGNO v2.0 codes performs these calculations on a gridded mesh on the plasma surface. The VMEC and PIES codes provide Fourier representations of the surface, and fields which are converted into cartesian coordinates. This provides a simple method for specifying the location and orientation of the diagnostics relative to the plasma. The user may specify the number of poloidal and toroidal (per field period) grid points to use in the representation of the surface. The user may also specify the methodology they wish to use for integrated quantities (such as the flux loops response $\int \vec{A} \cdot \vec{dl}$). Available methods include midpoint, Simpson and Bode. The user may also specify the number steps to take per line segment for the given integration method.

3. Results

The LHD provides an excellent geometry in which to test the DIAGNO v2.0 code. The helical plasma and 24 saddle type flux loops provide a highly three dimensional configuration (Figure 1). A vertical field ramp (with no plasma) provides a test of the codes ability to calculate diagnostic signals. Tests were conducted between the code and EXTENDER where the field was computed at various points in space for a given equilibrium. These tests show the codes agreed to within a few tenths of a percent. A series of equilibria at varying beta and net toroidal current were calculated for the LHD. These equilibria provide a a gauge of the sensitivity of the DIAGNO v2.0 code to plasma parameters.

A vertical field ramp was preformed in the LHD with no plasma present. This vertical field ramp moved the magnetic axis from a position of ~ 3.55 $[m]$ to ~ 3.60 $[m]$. The measured currents in the superconducting coils can be seen in Figure 2. Here the ramp in vertical field coils is clear around 2.0 $[s]$ into the shot. A direct calculation of the change in vacuum flux through each loop is preformed based on a mutual inductance calculated by DIAGNO v2.0 and plotted against the measured change in flux (Figure 3). The noise in the outer helical coil appears as a high frequency oscillation in the calculated signal. Still general agreement between measured and calculated change in flux can be seen in the plot. The other 18 saddle loops show similar features. The ability to calculate the diagnostic response due to field coil currents allows estimates of experimental noise attributed to fluctuations in the field coils. This is especially important in modern superconducting devices where changes in diagnostic signal are measured. This capability of the code also allows for calibration of flux loop signals.

As the DIAGNO v2.0 code was under development, tests were preformed to verify the integrity of the virtual casing principle. The code was debugged through comparisons with the EXTENDER code (which also employs a virtual casing principle, but does not calculate diagnostic response). The field produced by the plasma was evaluated at 29 points around the machine,

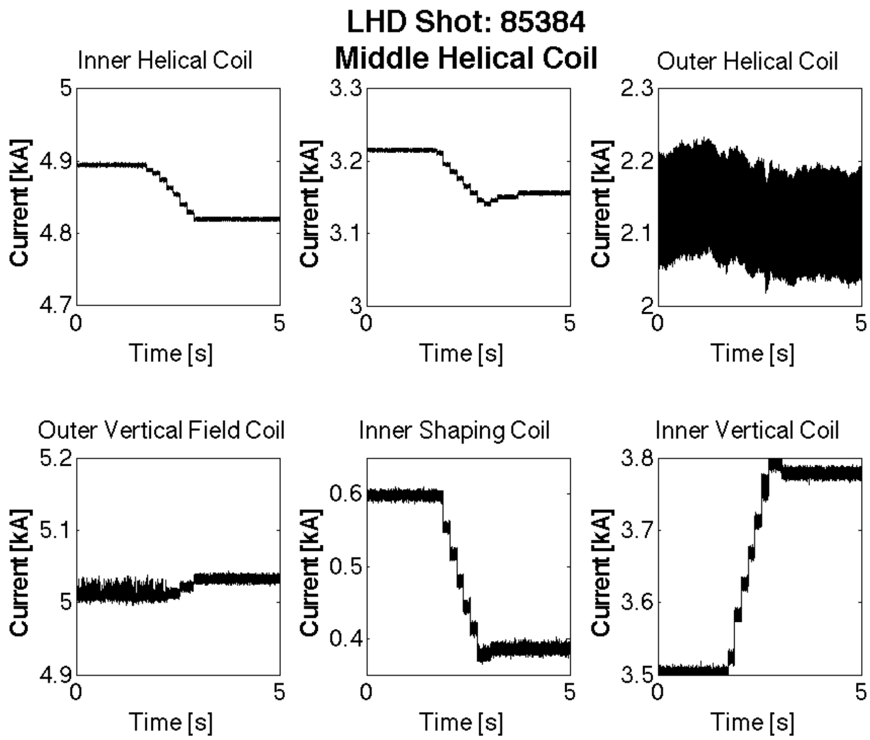

Figure 2: Vacuum shot coil currents on the LHD. The vertical field coils (PIV, PIS, POV) clearly show a change in current around 2.0 [s] into the shot. The helical coil shows some signs of ramping during this time as well. The outer helical coil shows a significant amount of noise when compared to the other coils. All plots have been scaled to reflect the same range in currents (~ 0.3 [kA]).

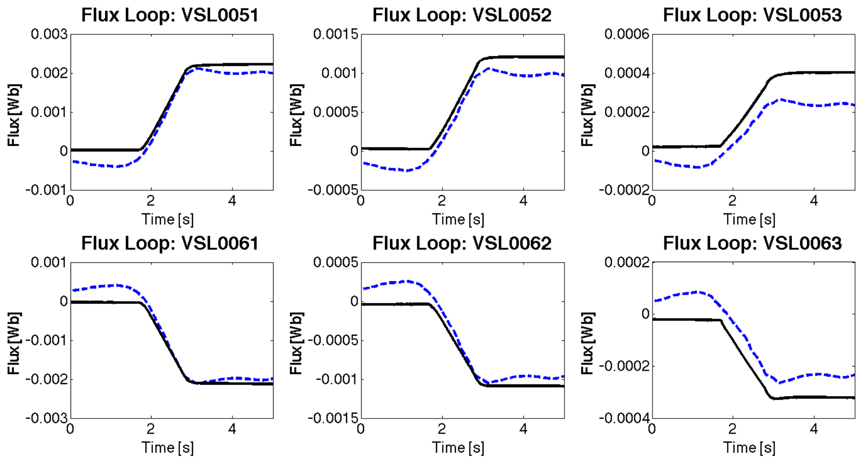

Figure 3: Change in flux through saddle loops on LHD for a calibration shot (ramp in field coils). Measured (solid line) and calculated flux (dashed line) through six of the 24 saddle loops on the LHD. The initial offset is attributed to smoothing of the calculated results. Only six of the 24 saddle loops are depicted here.

the codes agreed to machine accuracy. Comparisons between the original DIAGNO code (at zero net toroidal current) and the new version also show good agreement. Here the speedup associated with the virtual casing principle was clearly evident as the original code ran for 10 minutes and the new code runs in 20 seconds. This is associated with not having to calculate the total field on the surface of the plasma from the field coils.

Parameter sweeps were preformed of the plasma beta and net toroidal current. The first test involved calculating a series of equilibria on the LHD where the toroidal current was fixed at zero and plasma beta was varied. In the second test, the plasma beta on axis was held fixed ($\beta_{axis} \sim 0.01$) and the toroidal current was varied. In each case the profiles were assumed to be of a form $f\left(\Phi\right) = f_0 \left(1 - \Phi\right)^2$. In these these tests a high field coil energization were assumed (maximum current in each field coil). This provides a method for examining the contribution to diagnostic responses due to finite beta and net toroidal current effects. This is important as the highly three dimensional structure of the plasma and diagnostics makes a-priori assumptions difficult.

The plasma beta on axis β_{axis} was varied from vacuum to 0.05 and equilibria were calculated with VMEC. Figure 4 shows the variation in the pressure driven toroidal currents for two choices of β. Here we see that even for modest

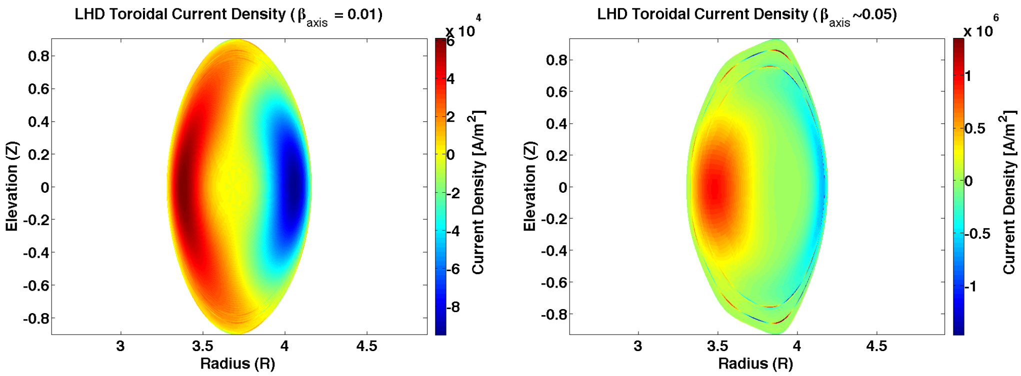

Figure 4: Toroidal current for $\beta_{axis} = 0.01$ (left) and $\beta_{axis} = 0.05$ (right). Pressure driven current densities on the order of 50 $[kA/m^2]$ are present for the $\beta_{axis} = 0.01$ case. The $\beta_{axis} = 0.05$ shows pressure driven current densities on the order of a 1 $[MA/m^2]$.

choices of plasma β current densities on the order of 50 $[kA/m^2]$ are generated. As $< \beta >$ is increased the inboard currents become more localized in the poloidal direction while the outboard currents become more radially localized. The presence of resonances in high β case suggest the presence of islands and stochastic regions. The plasma response in the saddle loops are indicated in figure 5. The simulated diagnostic responses are of the order of those measured during experimental campaigns. The response in the saddle loops is nearly linear for all loops. Comparisons against varying toroidal current at finite beta were made next.

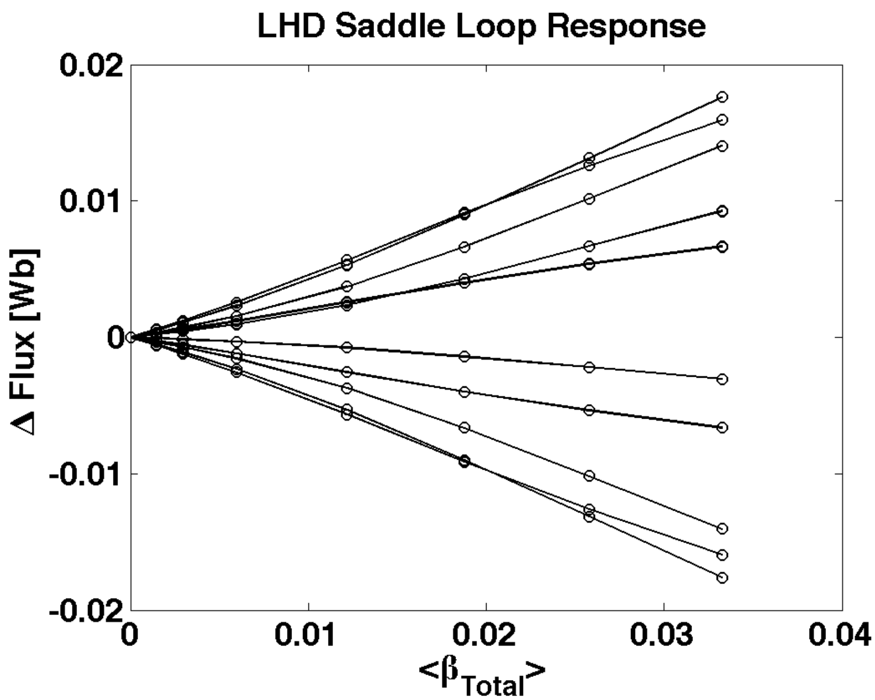

Figure 5: LHD saddle loop response to beta. Equilibria were calculated under the assumption of no net toroidal current and a high-field configuration. All loops show a sensitivity to changes in plasma beta. These changes are on the order of those recorded in experimental results.

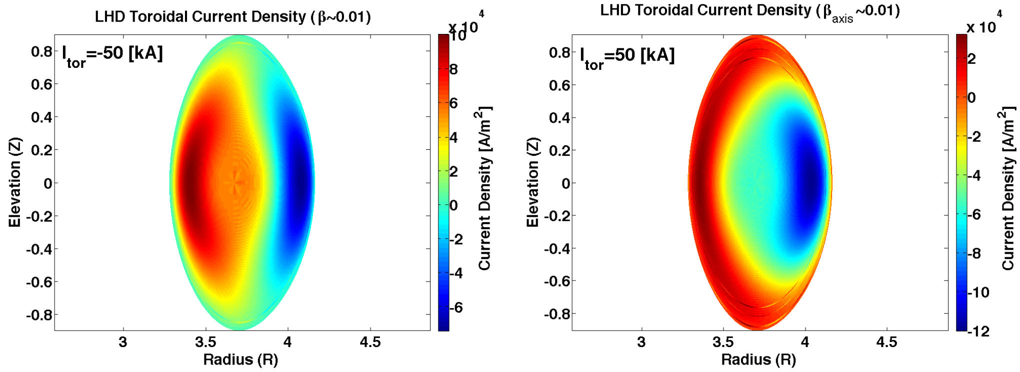

Figure 6: Torodial current for net toroidal current of -50 (left) and 50 $[kA]$ (right) at fixed beta. The distribution of toroidal current is clearly modified. Note that the convention for net toroidal current is opposite the convention plotted to a positive net toroidal current enhances negative currents. Note that positive values are in the negative ϕ direction.

The toroidal current was varied holding the pressure profile fixed ($\beta_{axis} = 0.1$). Figure 6 shows the effect of the toroidal current at -50 and 50 $[kA]$. The currents associated with finite beta are clearly being modified by the net toroidal current. Plots of the diagnostics response show signals (Figure 7) on the order of those generated by finite beta effects alone. This equates to the total toroidal current accounting for up to $\sim 40\%$ of the diagnostic signal at a plasma beta of ~ 0.01. This indicates that the saddle loops are much more sensitive to the pressure driven currents (and thus beta) than net toroidal currents in the plasma.

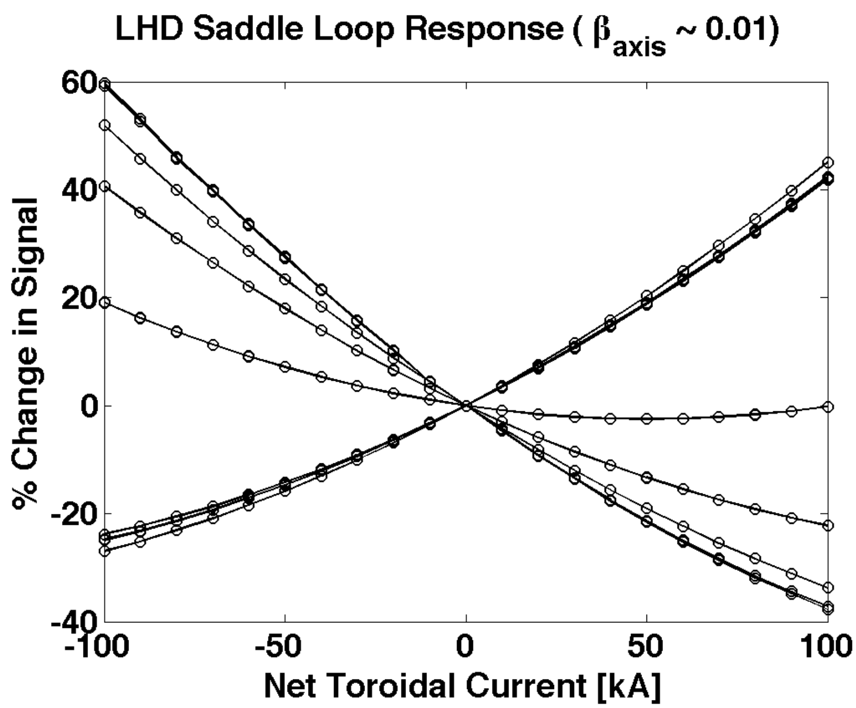

Figure 7: LHD saddle loop response to toroidal current (at finite beta). Values have had the finite beta signals removed. Equilibria were calculated under the assumption of a plasma beta on axis of 1%. Changes in diagnostic signal due to plasma currents are on the order of diagnostic signals due to finite beta effects.

4. Discussion

A new code, DIAGNO v2.0, has been developed for the calculation of magnetic diagnostic response to 3D equilibria. This code is capable of treating both equilibria generated by the VMEC and PIES codes through a virtual casing principle. This allows for calculation of the effects of islands and stochastic regions on magnetic signals (through PIES equilibria). This has the added benefit of significant speed enhancements for some magnetic configurations (as compared to the previous code). The utility of the code is further extended through it's ability to calculate magnetic signals directly from the field coils. The DIAGNO v2.0 code will allow for development of equilibrium reconstruction capabilities for magnetically confined plasmas with 3D fields.

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