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1.5D Quasilinear Model for Alpha Particle-TAE Interaction in ARIES ACT-I

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We study the TAE interaction with alpha particle fusion products in ARIES ACT-I using the 1.5D quasilinear model. 1.5D uses linear analytic expressions for growth and damping rates of TAE modes evaluated using TRANSP profiles to calculates the relaxation of α pressure profiles. NOVA-K simulations are conducted to validate the analytic dependancies of the rates, and to normalize their absolute value. The low dimensionality of the model permits calculating loss diagrams in large parameter spaces.

I. INTRODUCTION

The Advanced Reactor Innovation and Evaluation Study (ARIES) is a research program aimed at designing optimized reactor concepts accessible to the general consumer.
ARIES explores a wide range of parameters for designing viable operating plasmas for high fusion performance tokamaks [1]. In ARIES designs, the fusion product α particles are assumed abundant and their confinement is crucial for the success of the design. However, α particles' velocities are comparable to the Alfvenic velocity in most of ARIES concepts. Depending on the existing damping mechanisms from the background plasma, free energy from the radial gradient in the pressure profiles of α particles could be enough to drive the toroidally induced Alfvenic Eigenmodes, TAE modes, unstable. α particles are in turns transported by these destabilized TAE modes. Large scale transport due to overlapped TAE modes is possible and may induce α-particle loss which degrades the plasma performance and damages the first wall. The possibility of this large scale transport makes it a pressing issue to understand α-TAE interaction and have an estimate of the influence this interaction could have on the confinement of α particles.
We focus in this study on ARIES ACT-I designs. They are D-T plasmas at ion temperatures around 30 keV and quasi-neutrality $\sigma = (n_D + n_T)/n_e \approx 0.7$ with plasma $\beta \approx 0.05 - 0.16$. The major axis, $R = 5.6$ m; minor axis, $a = 1.38$ m, and on-axis magnetic field, $B \approx 5.4$ T.

II. 1.5D QUASILINEAR MODEL

We use the 1.5D QL model, proposed in [6] and implemented and validated in [2],[3] to get an estimate of the effect of TAE interaction with fusion product α particles. The model is based on linear expressions[7, 13] for growth and damping rates of Toroidally induced Alfvenic Eignemodes, TAEs, to find the critical gradient of the pressure profiles that would result in marginal stability of the TAE modes.
TAE modes are driven by free energy from the α particle pressure profile, $\gamma \propto \omega\partial\beta/\partial E + n\partial\beta/\partial P_\phi$ where ω is the TAE frequency, and n its toroidal mode number. (P_ϕ, E) is the α particle canonical momentum and energy phase space variables. Since $P_\phi \propto -r$, the negative gradient in r results in a positive gradient in P_ϕ which is a destabilizing contribution. $P_\phi\omega/(nE) << 1$ in most fusion devices with burning plasmas, so we neglect the $\partial\beta/\partial E$ contribution, and the TAE growth rate is proportional to the radial gradient in α-particle pressure profile $\gamma_\alpha \propto \partial\beta_\alpha/\partial r$.
There are various damping mechanisms[4, 8, 14, 17] due to the background plasma, mainly, electron collisional damping, ion Landau damping and radiative damping. TAE modes are destabilized if the instantaneous growth rate γ_α is larger than the sum of the damping rates, $\gamma_{dmp} = \gamma_{eColl} + \gamma_{iL} + \gamma_{rad}$. In Quasilinear theory[15, 18, 19], the existence of unstable modes result in diffusion of α-particles which diminishes the slope to modify the instantaneous growth rate. The 1.5D QL model assumes that the α-particles continue to undergo transport to the point the TAE modes achieve marginal stability where the growth rate is equal to the damping rates. Writing the growth rate as $\gamma_\alpha \equiv \gamma'_\alpha \partial\beta_\alpha/\partial r$, the condition on the pressure gradient in marginal stability becomes

$$\frac{\partial\beta_\alpha^{crt}}{\partial r} = \frac{\gamma_{dmp}}{\gamma'_\alpha} \tag{1}$$

The unstable region, $r_{unS} = [r_-, r_+]$ is where the initial gradient $\partial\beta_\alpha^{ini}/\partial r > \partial\beta_\alpha^{crt}/\partial r$. We start the process of finding the relaxed profile by assuming the gradient in region r_{unS} to be the critical gradient. To insure continuity and conservation of particles, the region boundaries need to relax to $r_{rel} = [r_a, r_b]$. We start with $r_{rel} \equiv r_{unS}$, then r_{rel} is iteratively expanded and the profile integrated using $\partial\beta_\alpha^{crt}/\partial r$ in r_{rel}. The region continues to expand until the resulting β_α^{rel} satisfies conditions of continuity and conservation of particles in addition to having the critical gradient over t least the unstable, $r_{unS} \subset r_{rel}$. This results in relaxation of the pressure profile beyond the initially unstable region. The distribution relaxes in the phase space region where particles resonantly interact with the TAE modes. This is only a percentage, η, of phase space where $v_{||} < v_{\alpha 0}$. Using the expression in [16], this is $\eta = (v_{\alpha 0} - v_{||})v_{||}/v_{\alpha 0}^2 \approx 25\%$ for isotropic α

particles, which results in a relaxed distribution function

$$\beta_\alpha = (1-\eta)\beta_\alpha^{ini} + \eta\beta_\alpha^{rel} \tag{2}$$

The region, r_{rel} can keep expanding all the way beyond the last closed flux surface in attempt to conserve particles. In such cases, particles are lost and the restriction on conservation of particles is released. The iteration stops and the integrated relaxed pressure profile will have less particles. The loss fraction is

$$loss = \frac{\int \mathrm{d}r(\beta_{rel}(r) - \beta_{ini}(r))}{\int \mathrm{d}r\beta_{ini}(r)} \tag{3}$$

III. CASE STUDY

We use ARIES run 1000A53 as a quintessential case study for ARIES ACT-I to demonstrate the 1.5D QL model used to make a parameter space study of the TAE induced α particles loss.

A. 1.5D QL Model

Using TRANSP profiles for ion temperature, T_i, plasma beta, β_p and β_α we calculate the linear growth and damping rates at all r. We assume that there could exist a mode at each radial position, and the toroidal mode number of the most unstable mode would be such that $k_\perp\rho_\alpha \approx 1$ where $k_\perp$ is the perpendicular wave number and ρ_α is the larmor radius of α-particles. This results in $nq^2\rho_\alpha/r \approx 1$ where q is the safety factor, r is radial position, and n is the toroidal mode number.

The linear expressions for the growth and damping rates are

$$\frac{\gamma_\alpha}{\omega} = -\frac{5\pi}{2}q^2 r\frac{\partial\beta_\alpha}{\partial r}x_A(1-x_A^2) \tag{4}$$

$$\frac{\gamma_i}{\omega} = -\frac{q^2\sigma\sqrt{\pi}\beta_p}{2(1+\sigma)}x_i^5 \mathrm{e}^{-x_i^2} \tag{5}$$

$$\gamma_{rad} = -3\left(\frac{\rho_s sm(m+1)}{\sqrt{2}r(2m+1)}\right)^{2/3} \tag{6}$$

$x_A = v_A/v_{\alpha 0}$ and $x_i = v_A/v_i$ where $v_A = B/\sqrt{4\pi n_{iD}m_{iT} + n_{iT}m_{iT}}$ is the alfvenic velocity where $n_{iD,T}$ and $m_{iD,T}$ are the deuterium and tritium densities and masses respectively, $v_{\alpha 0}$ is the alpha birth velocity and $v_i = \sqrt{2T_i/m_i}$ is the ion velocity. s is the local shear, m is the poloidal mode number, and $\rho_s = c/\Omega_{\alpha c}$ where c is the speed of sound, and $\Omega_{\alpha c}$ is the α gyrofrequency.

B. NOVA-K Simulation

To verify the analytically computed rates and to normalize the values in case of discrepancy, we run NOVA-K [5] to compute the linear growth and damping rates at $t = 600$. We choose two modes located at two different radial positions. The mode structures computed by NOVA, fig(1), are of toroidal mode number $n = 9$ at $r = 0.4a$ and $n = 5$ at $r = 0.6a$ with frequenies $\omega = 1.06$ and $\omega = 0.89$ respectively.

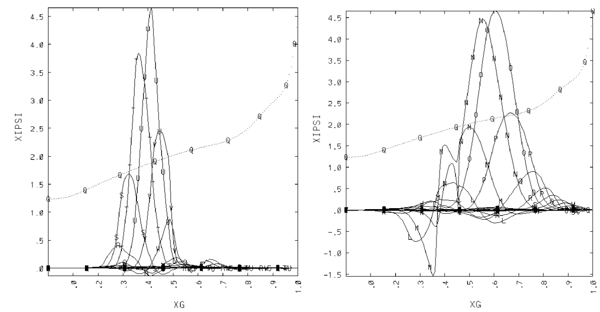

FIG. 1: The mode structure of TAE modes are computed by NOVA code as a function of $XG = \sqrt{\psi/\psi_0}$ where ψ is the normalized poloidal flux. The left figure is the mode with ($n = 9, \omega = 0.89$) localized around $r/a = 0.4$ and to the right, the mode ($n = 5, \omega = 0.89$) localized around $r/a = 0.6$

For $t = 600$

mode location	γ_{growth}/ω	$\gamma_i/\omega + \gamma_{iL}/\omega$	γ_{rad}/ω
$r/a = 0.4$	19%	5%	6%
$r/a = 0.6$	10%	9%	2%

We use NOVA-K results for normalizing the linear expressions, we get the following results at the given times for discharge 1000A53. In fig(2) we see the radial dependencies of the rates and resulting critical β gradient in comparison to the initial gradient. The resulting pressure profile relaxes beyond the last closed flux surface which results in losses. 4% of α particles were lost due to the interaction with the TAE modes.

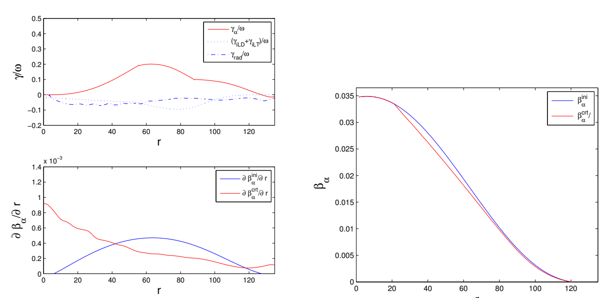

FIG. 2: The figure to the top left corner depicts the radial dependency of the growth rate (solid red), and the damping rates, (dotted and dashed blue). In the bottom left corner, the critical gradient in β_α (red) is represented in comparison to its the initial gradient (blue). The right figure depicts the resulting relaxed profile β_α^{rel} (red) in comparison to the initial profile, β_α^{ini} (blue)

We analyze the shot using TRANSP files at the rest of the time $t = 250$, 400, 600, 800, and 1190 using the same normalization coefficients resulting from NOVA simulations at $t = 600$. Amongst the damping and drive mechanisms, only the radiative damping is significantly modified with time. We present in fig(3) the resulting relaxed profiles and the change in radiative damping.

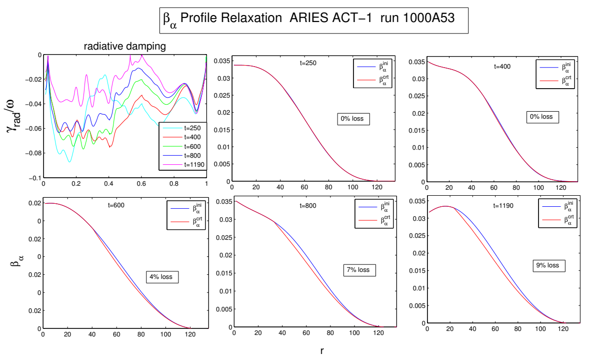

FIG. 3: The radiative damping change with time is presented in the top left corner. Following it are the results of 1.5D on the relaxation of the α particle profiles with time where the TRANSP provided β_α profiles are in blue and the 1.5D relaxed profiles in red.

IV. ALPHA PARTICLE LOSS DIAGRAM

Since ARIES designs cover a range of parameter, 1.5D QL model lends itself as a valuable tool for the analysis of the effect of TAE interaction with fusion product alpha particles over these large parameter spaces. The relaxation of alpha pressure profiles and resulting possible losses are computed using analytic estimates of the profiles. We show that the analytic estimates are reliable in comparison with the self-consistent computations of TRANSP [10]. Using the analytic profiles, we can readily apply 1.5D model to find the loss diagram over a wide parameter space $(\beta_{p0}, T)i0)$. For evaluating the growth ad damping rates, we still need the ion density profiles to evaluate $x_i = v_A/v_i$ and $x_A = v_A/v_{\alpha 0}$. However, we use the approximation $x_{iD} \approx \sqrt{(1+\sigma)/(9(1+\sigma/4)\beta_p)}$ [8] resulting in $x_{iT} \approx x_{iD}\sqrt{3/2}\ x_A \approx 0.00226 x_{iD}\sqrt{T}$ where T is in eV.
We use quadratic approximations for the ion temperature and plasma pressure profiles, $T(r) = T_0(1-r^2)$, and $\beta_p(r) = \beta_{p0}(1-r^2)$ respectively. Knowing the temperature and pressure profiles allows for approximating the resulting alpha particles fusion product beta profiles

$$\frac{\beta_\alpha}{\beta_p} = \frac{8 n_D n_T}{n_e^2(1+\sigma)} \frac{< \sigma v > n_e \tau_{se} \mathcal{E}_{\alpha 0}}{12T} \tag{7}$$

where σ is the quasi-neutrality coefficient. We use the expression from [11] for $\xi(T) \equiv < \sigma v >$, the cross section of the thermonuclear D-T reaction as a function of temperature.

$$\xi(T) = \frac{V_0}{T^{2/3}} \left[\frac{T - T_0}{1 + d(T-T_0)^2} + g \exp -\mu T^\nu \right] \exp - \frac{C}{T^{1/3}} \tag{8}$$

where the constants are approximated for the various temperature ranges where $T < 50keV$ is the range we use.
τ_{se} is the energy slowing down time of alpha particles on the background plasma due to coulomb collisions with electrons and ions. Since $v_{Ti} < v_\alpha < v_{Te}$ the slow down time is approximated [11], [8] by $n_e \tau_s \approx 4\text{x}10^{12} T^{2/3}$ where T is expressed in keV.
Using $n_{iD} \approx n_{iT}$ the expression for β_α is then simplified to

$$\beta_\alpha = \beta_p \frac{2\times 10^{12}\sigma^2}{3T(1+\sigma)} \xi(T) T^{3/2} \mathcal{E}_{\alpha 0} \tag{9}$$

We compare in fig(4) the profiles calculated using the analytic expressions to the those in TRASNP for a given run. We use the T_0 and β_{p0} from TRASNP in the analytic expressions for $T(r)$ and $\beta_p(r)$ to calculate the resultant $\beta_\alpha(r)$. This remarkable agreement gives us confidence in

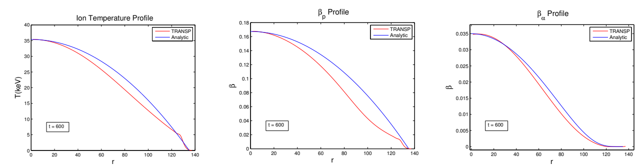

FIG. 4: The ion temperature profiles, a, plasma beta profile, b, and the alpha fusion product beta profiles, c, using TRANSP files (red) vs the analytic profiles computed using quadratic expression for T and β_p, and the analytic expression, eq(9), for β_α.

using the analytic equations to reliably compute the incurred losses over a vast parameter space.
Using 1.5D QL for the various values of T_0 and β_{p0} we can find the losses, fig(5) resulting from the TAE interaction with the α-particles. Note that there could be cases where the TAE modes are linearly unstable and relaxation of the profile can occur without any losses. The region in parameter space where TAE modes are linear unstable is also depicted in fig(5). Note that there is a region of space where TAEs are unstable but no losses incur. That is the case in which relaxation occurs such that the relaxation region r_{rel} is not extended beyond the last closed flux surface. This parameter space diagram is computed without accounting for Radiative damping which could shift the diagram significantly. For the special case of discharge 1000A53, the parameters are such that $T_{i0} \approx 35$ keV and $\beta_{p0} \approx 0.16$.
This shot has been analyzed using TRANSP files and

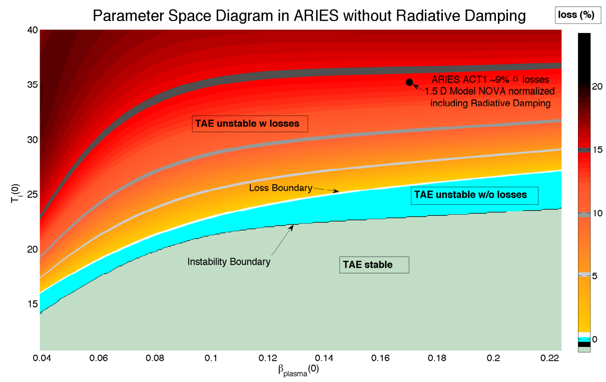

FIG. 5: A depiction of the loss fraction in (β_p, T_i) space. The light green region of parameter space bounded by a black line is where TAE modes are stable. The space depicted in cyan is where TAE modes are destabilized, however α particles are only redistributed without being lost. Beyond the yellow line, losses start incurring where the percentage of lost particles corresponds to that in the color bar.

NOVA-K simulations as discussed in sec(III) rakes into consideration all the damping mechanisms. However, we rerun the code without accounting for radiative damping to com are it to the diagram result. Since the other mechanisms are dependent predominantly on T_i, β_α, and β_p which are similar for all times, the losses are consistently around 15% which is in agreement with the diagram.

V. CONCLUSION

This is a first step in the analysis of TAE stability and α-particle transport in ARIES ACT-I. This analysis gives a good ballpark picture of the possible TAE-α interaction and shows confidence in α particle confinement in ARIES ACT-1 designs. However, there is a need for more rigorous analysis of the damping mechanisms especially radiative damping which is the key mechanism preventing α particles from being lost according to the 1.5D model.
Currently, a line broadening quasilinear model, LBQ2D, is being developed that analyzes the α-particles diffusion in phase space as TAE modes evolves self-consistenly. LBQ2D handles regimes in which modes are isolated as well as regimes in which multiple modes overlap, and it accounts for diffusion in both P_ϕ and E phase space. While LBQ2D captures the drive mechanisms and resulting diffusion more accurately and self-consistently then 1.5D, it does not compute the damping mechanisms and relies on existing expressions which are verified, but are in need of further development such as the radiative damping.

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