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Prepared for the U.S. Department of Energy under Contract DE-AC02-09CH11466.

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# Enhanced efficiency of internal combustion engines by employing spinning gas

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(Dated: February 14, 2014)

The efficiency of the internal combustion engine might be enhanced by employing spinning gas. A gas spinning at near sonic velocities has an effectively higher heat capacity, which allows practical fuel cycles, which are far from the Carnot efficiency, to approach more closely the Carnot efficiency. A gain in fuel efficiency of several percent is shown to be theoretically possible for the Otto and Diesel cycles. The use of a flywheel, in principle, could produce even greater increases in the efficiency.

PACS numbers: 05.70.Ce, 47.55.Ca, 07.20.Pe

*Introduction:* Optimizing the internal combustion engine to achieve the highest possible fuel efficiency can be approached both from a theoretical perspective and from a practical perspective [1]. From the practical perspective, which has attracted the most attention, research has focused on the optimization of the irreversible processes that occur in the working engine, by considering finite time thermodynamics [2] and irreversible thermodynamics [3]. These processes include friction losses [4], inhomogeneous combustion and heat transfer to the wall [5, 6], optimal piston trajectory [7], and other non-ideal effects in combusting gas [8]. From a theoretical perspective, equilibrium thermodynamics places upper bounds on efficiencies, which in practice are not nearly reached.

It is predicted here how higher theoretical limits might be reached in practical cycles by exploiting the sensitivity of the efficiency of practical fuel cycles to the heat capacity of the working gas. Recently, it was shown that the spinning of a gas equips it with an effectively higher, spin-dependent heat capacity, with the largest heat capacity appearing for sonic spinning speeds [9]. It turns out, as we show here, that significant improvements in efficiency might be possible by using this effect in the internal combustion engine.

Specifically, it is proposed here to introduce finite angular momentum to the working gas, namely by spinning it around the axis of the cylinder. The higher heat capacities may then be exploited to give higher efficiency in practical fuel cycles, like the Otto and Diesel cycles, where, as we show, a gain in fuel efficiency of as much as several percent is theoretically possible. The new theoretical limits rely only upon equilibrium thermodynamics of spinning gases, and are achieved independent of the details of finite time irreversible thermodynamics.

It is assumed here that the working fluids are ideal, Boltzmann gases and that the chemical reactions of combustion do not change the gas constituents significantly. The gases may be compressed axially in a cylindrical container, like in a typical engine cycle. The only difference is that the gases may be spinning around the axis of the cylinder. The gas angular momentum is assumed to be conserved on the time scale of the compression; in other words, the cylinder is assumed to be frictionless. The compression or expansion cycles, however, are assumed to be slow enough that equilibrium thermodynamics pre-

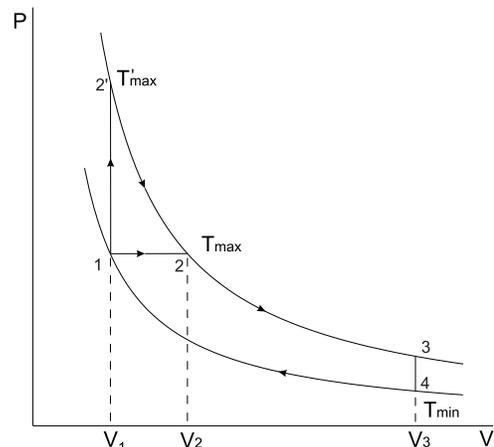


FIG. 1: Otto cycle  $1 \rightarrow 2' \rightarrow 3 \rightarrow 4$  and Diesel cycle  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ . Heat is transferred at  $1 \rightarrow 2'$  or  $1 \rightarrow 2$  phase.

vails, under the constraint imposed by the conservation of the angular momentum. Note that the enhanced heat capacity utilized here arises from this constraint on the collective motion of the gas constituents, rather than the spin properties of individual atoms.

*Engine efficiency:* Consider two practical cycles for engines, namely the Otto cycle and the Diesel cycle. The P-V diagrams of these cycles are shown in Fig. 1. The Otto cycle consists of adiabatic compression and expansion processes, separated by ignition and rejection of heat processes at constant volume. The Otto cycle efficiency depends only on the volumetric compression ratio  $n = V_{\max}/V_{\min}$ , and is given by

$$\eta_o = 1 - n^{1-\gamma}, \quad (1)$$

where  $\gamma = c_p/c_v$  is the specific heat ratio. In the Diesel cycle, the heating occurs at constant pressure, rather than constant volume, as in the Otto cycle. If the ratio of the volumes after heating and before heating is  $p = V_2/V_1$  and the total volume compression ratio is  $n = V_{\max}/V_{\min}$ , the Diesel cycle efficiency may be written as

$$\eta_d = 1 - \frac{1}{\gamma n^{\gamma-1}} \frac{p^\gamma - 1}{p - 1}. \quad (2)$$

Depending on what constraints are imposed, any of these types of engines might be the most efficient. Given say maximum and minimum volumes, the Otto cycle would be most fuel efficient, even more so than the Carnot cycle.

However, constraints on the temperature appear to be the most fundamental from a practical viewpoint. Constraints on the volume are likely less important, since usually there is ample room for the engine. Constraints on pressure might be circumvented by the use of compressors. On the other hand, material properties limit temperature on the high side; while the ambient temperature marks the low temperature limit. Given two limiting temperatures, namely a maximum temperature  $T_{\max}$  and a minimum temperature  $T_{\min}$ , the Carnot cycle gives, of course, the optimum fuel efficiency,  $\eta_c = 1 - T_{\min}/T_{\max}$ .

In practice though, the Carnot cycle is impossible, because, first, it contains isothermal processes that are not implementable, and second, it requires a heat reservoir at  $T_{\max}$  that is not present in real engines. Thus, consider instead the Otto and Diesel cycles, but constrained by maximum and minimum temperatures. To render the efficiency of these cycles, Eq. (1) and Eq. (2), in terms of the temperature extrema, introduce the ratio of minimum and maximum temperatures,  $\delta = T_{\min}/T_{\max}$ , and the ratio of total heat per particle and maximum temperature,  $q = Q/NT_{\max}$ , so that the efficiencies of the Otto and Diesel cycles can be put as

$$\eta_o = 1 - \frac{\delta}{1 - q/c_v}, \quad (3)$$

$$\eta_d = 1 - \frac{\delta c_v}{q} \left( \frac{1}{(1 - q/c_p)^\gamma} - 1 \right), \quad (4)$$

respectively. Note that, for  $q$  large enough, a singularity appears in the denominators, indicating that such processes are not feasible, namely, that more heat is introduced than can be accommodated by the temperature difference. For small  $q$ , the Otto cycle efficiency can be approximated as  $\eta_o \approx 1 - \delta - q\delta/c_v$  and the Diesel cycle efficiency as  $\eta_d \approx 1 - \delta - (q\delta/c_v)(\gamma + 1)/2\gamma$ , so that it can be seen that, as  $q \rightarrow 0$ , the Diesel cycle is more efficient than the Otto cycle for all temperature ratios.

*Spinning gas:* Consider now the effect of spinning the working gas in each of these thermal cycles. Two types of compression may now be distinguished, axial and perpendicular, since a centrifugal force now acts on the gas. However, here only the longitudinal compression (along the axis of the spinning) will be considered, since radial compression is very hard to realize practically.

The thermodynamic properties of spinning gas are captured entirely by one parameter that captures the spinning, or what we call the *spinning parameter*  $\varphi = m\omega^2 r_0^2/2T$ , which measures the spinning energy compared to the thermal energy [9]. Here  $m$  is the mass of gas molecule,  $r_0$  is radius of the cylinder,  $\omega$  is angular frequency, and  $T$  is the gas temperature. For negligible

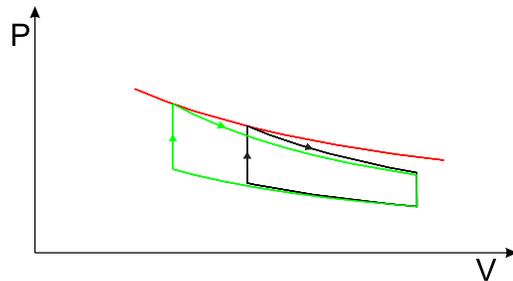


FIG. 2: Modification of Otto cycle for spinning gas. Red curve is the temperature constraint. Black curve denotes a non-spinning case; green curve denotes spinning case.

friction losses over a thermal cycle, the angular momentum of the gas, given by

$$M_g = Nm r_0^2 \omega A(\varphi), \quad (5)$$

is conserved, where the function  $A(\varphi)$ ,

$$A(\varphi) = \frac{e^\varphi(\varphi - 1) + 1}{\varphi(e^\varphi - 1)} \quad (6)$$

is the normalized moment of inertia of the gas. It changes from  $1/2$  to  $1$  as  $\varphi$  goes from  $0$  to  $\infty$ . The gas energy is given by

$$E = c_v NT + \omega M_g/2, \quad (7)$$

where the second term denotes the energy of rotation.

The physical picture is as follows: Rotation flings the gas molecules to the cylinder walls, an effect counteracted by high temperature. Under compression, the gas adiabatically heats up, forcing molecules away from the walls, thereby decreasing the moment of inertia  $A(\varphi)$ . Since angular momentum is conserved, the angular velocity must increase, as does the energy of rotation. This effect can be described as a rotation-dependent heat capacity that now goes from  $c_v$  to  $c_v + B(\varphi)$ , where  $B(\varphi)$  is a smooth *compression function* that goes from  $0$  to  $1$  as  $\varphi$  goes from  $0$  to  $\infty$  [9]. For small  $\varphi$ , the system behavior is very close to the non-spinning case; only for  $\varphi \geq 1$  is the difference noticeable. The parameter  $\varphi$  changes under compression or heating of the gas, but the change is modest. Thus, under axial compression, in the limit of high  $\varphi$ , the effect of rotation is to increase the specific heat  $c_v$  by  $1$ .

Note that, if constrained by a fixed compression ratio, it is inefficient to compress rotating gas axially in the Otto cycle, where the efficiency increases with  $\gamma$ . For example, for a monatomic gas with  $c_v = 3/2$  and  $\gamma = 5/3$ , the specific heat increases to  $5/2$ , meaning that  $\gamma = 7/5$  in the limit of supersonic spinning. For compression ratio  $n = 2$ , the efficiency  $\eta \approx 0.24$  for the spinning gas is less than the efficiency  $\eta \approx 0.37$  for non-spinning gas. In contrast, if constrained by a fixed temperature ratio, as seen from Eq. (3), the Otto cycle is more efficient under

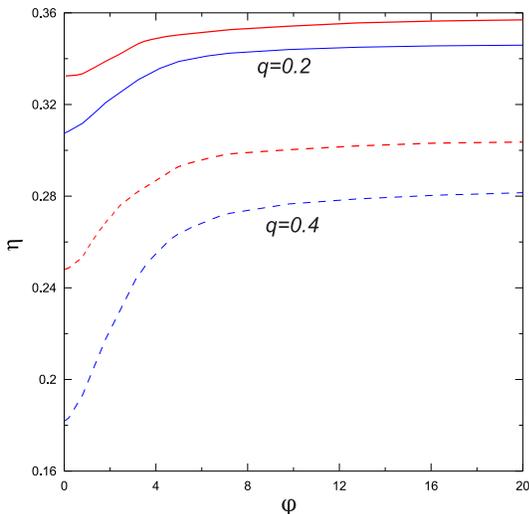


FIG. 3: Efficiency of Otto and Diesel cycles as function of spinning parameter  $\varphi$  at maximum temperature of the cycle. Solid line is for  $q = 0.2$ , dashed line for  $q = 0.4$ , red color for Diesel cycle, blue color for Otto cycle.

spinning by

$$\hat{\eta}_o - \eta_o \approx q\delta/c_v c_p. \quad (8)$$

This difference can be of the order of several percent, a very significant improvement compared to typically realized engine efficiencies of the order of 20-30 percent. Fig. 2 demonstrates how the Otto cycle, under spinning, traces a modified, larger area, P-V curve. The larger heat capacity accommodates the maximum temperature constraint, while the volume ratio increases in order to increase the cycle efficiency.

Fig. 3 shows how the efficiency increases with the spinning parameter  $\varphi$ . The efficiency increase depends on the amount of heat transferred to the system  $q$ , but the efficiency always increases with  $\varphi$  for all  $q$ . The efficiencies are higher for lower values of  $q$ , since lower  $q$  signifies a cycle closer to the ideal Carnot cycle. However, to overcome fixed inefficiencies in real devices,  $q$  is generally designed to be finite. Hence, it is particularly noteworthy that the greater efficiency improvement through spinning is available specifically for higher  $q$ .

Since the Diesel cycle begins with somewhat better efficiency, particularly for  $q$  small, there is somewhat less room for improvement in using the spinning gas. However, as can be seen from Eq. (4), like for the Otto cycle, the efficiency grows with  $\varphi$  up to saturation. For either cycle, strong gas rotation  $\varphi \geq 1$  is necessary to achieve significant improvement in the efficiency.

*Initiating the Spinning:* The key technical issue is how to introduce angular momentum to the system. One possibility is to bleed compressed gas into the cylinder along the cylinder wall in the tangential direction, so that the incoming gas follows the side cylinder wall. This initiation of the spin is similar to techniques used in vortex

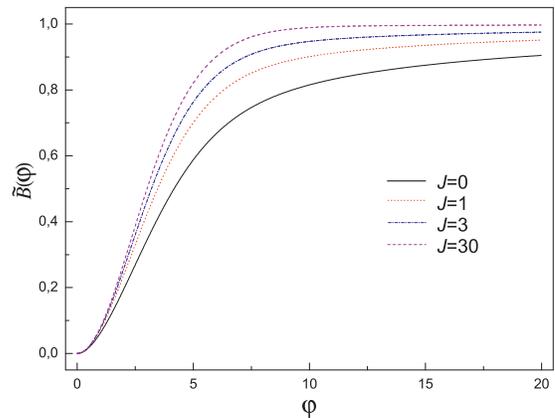


FIG. 4: Dependence of compression function  $\tilde{B}$  of  $\varphi$ .

tubes [10, 11]. The initial gas compression might be done, for example, by use of turbo compressors, similar to that installed in many modern engines.

*Flywheel Control:* Angular momentum might also be introduced through a flywheel. Suppose then an ideal, frictionless flywheel, with blades rotating inside the cylinder all the time, even between thermal cycles, such that the gas and the flywheel equilibrate to the same angular rotation velocity. The flywheel exchanges with the gas the mechanical energy of rotation. Suppose the flywheel has moment of inertia  $I$ , so that the total angular momentum becomes

$$M_{\text{tot}} = I\omega + M_g, \quad (9)$$

where  $M_g$  is given by Eq. (5). The flywheel kinetic energy can then be added to the gas energy given by Eq. (7) to give the total energy,

$$E_{\text{tot}} = c_v NT + M_{\text{tot}}\omega/2 = I\omega^2/2 + c_v NT + Nm\omega^2 r_0^2 A(\varphi)/2. \quad (10)$$

Using now Eq. (10) together with Eq. (9), and assuming conservation of angular momentum, a generalized compression function  $B$  can be found

$$\tilde{B} = \frac{\varphi^2 A(\varphi) H(\varphi) (1 + J/A(\varphi))}{J/A(\varphi) + 1 + 2\varphi H(\varphi)}, \quad (11)$$

where the dimensionless parameter,  $J = I/Nmr_0^2$ , measures the moment of inertia of the flywheel compared to that of the spinning gas. In the limit  $J \rightarrow 0$ , the compression function reduces to the gas-only compression function  $B$ , found previously [9]. Fig. 4 shows how the compression function  $\tilde{B}(\varphi)$  increases with  $J$ , thus giving greater added heat capacity at smaller, presumably more easily obtainable, spinning parameters  $\varphi$ .

Now consider what happens if the flywheel is given angular velocity  $\omega_1$  while the gas has  $\omega_0$ . Equilibrium is established at the final angular velocity  $\omega$ , with gas

temperature changing from  $T_0$  to  $T$ , where  $\omega$  and  $T$  may be found using Eqs. (9) and (10), to get

$$M_{\text{tot}} = I\omega_1 + Nmr_0^2 A(\varphi_0)\omega_0 = \omega(I + Nmr_0^2 A(\varphi)), \quad (12)$$

$$c_v NT_0 + \frac{I\omega_1^2}{2} + \frac{Nmr_0^2 A(\varphi_0)\omega_0^2}{2} = c_v NT + \frac{I\omega^2}{2} + \frac{Nmr_0^2 A(\varphi)\omega^2}{2}. \quad (13)$$

It is more convenient to express quantities in terms of a new spinning parameter  $\varphi$  instead of the frequency  $\omega$ . After some algebra,  $\varphi$  at equilibrium may be written as

$$\varphi = \varphi_0 \frac{(JR + A(\varphi_0))^2 (J + A(\varphi))^{-2}}{\left[1 + \frac{\varphi_0}{c_v} \left(JR^2 + A(\varphi_0) - \frac{(JR + A(\varphi_0))^2}{J + A(\varphi)}\right)\right]}, \quad (14)$$

where  $\varphi_0 = m\omega_0^2 r_0^2 / 2T_0$  and  $R = \omega_1 / \omega_0$ . For  $T$  we have

$$T = \frac{T_0 \varphi_0}{\varphi} \left( \frac{JR + A(\varphi_0)}{J + A(\varphi)} \right)^2. \quad (15)$$

Note that the mechanical energy required to change the angular velocity of the flywheel from  $\omega_1$  to  $\omega_0$  is given by

$$\Delta E = \frac{I\omega_0^2}{2} - \frac{I\omega_1^2}{2} = JNT_0\varphi_0(1 - R^2). \quad (16)$$

Eqs. (14)–(15) describe how gas is spun up or slowed down by the flywheel. In effect, these equations describe removing the flywheel from the gas, changing its angular velocity from  $\omega_0$  to  $\omega_1$ , then again making contact with the gas until a new equilibrium is reached. In the limit of differentially small changes,  $dT = T - T_0$  and  $d\omega = \omega - \omega_0$ , and hence also  $d\varphi$  and  $dE$ , differential equations for  $T$  and  $E$  may be obtained:

$$\frac{dT}{d\varphi} = \frac{T}{\varphi} \frac{\varphi^2 A'(\varphi)}{c_v}, \quad (17)$$

$$\frac{dE/d\varphi}{NJT} = \left(1 + \frac{A(\varphi)}{J}\right) \left(1 + \frac{\varphi^2 A'(\varphi)}{c_v}\right) + \frac{2\varphi A'(\varphi)}{J}. \quad (18)$$

Note two important properties of this process. First, from the nature of differential equations, it is immediately seen that it is reversible: gas heats up when it is spun up and cools down when the rotation is slowed. Second, since no external heat was transferred to the gas, this process is adiabatic, hence it must be commutative with adiabatic compression and expansion, otherwise the second law of thermodynamics would be violated.

The spinning gas thermal cycle thus can operate as follows. First, the flywheel produces some initial rotation. The gas is then compressed and heated. The fuel is then burned and the gas expands. Lastly, the gas is

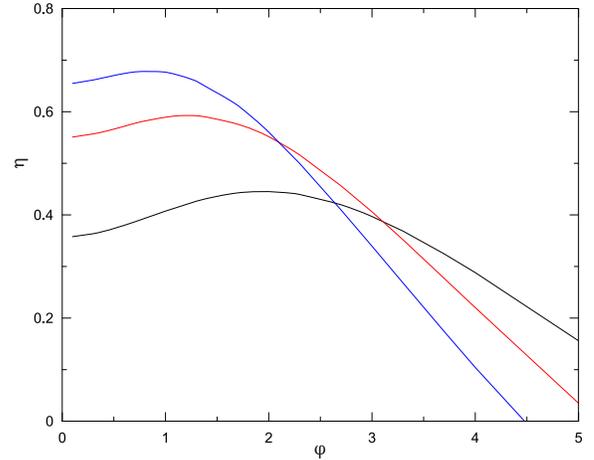


FIG. 5: Efficiency of Otto cycle for vs.  $\varphi$  for  $J = 1$ . Black:  $\delta = 4/3$ ,  $q = 1/2$ , red:  $\delta = 5/3$ ,  $q = 2/5$ , blue:  $\delta = 2$ ,  $q = 1/3$ .

slowed down by the flywheel, which cools it further. The total amount of work done in the cycle is the sum of two adiabatic compressions and two gas rotations with the flywheel. Note that, after the first stage of spinning injection, the gas heats up, thereby increasing the minimum temperature from where the adiabatic compression starts. Since the maximum temperature is constrained, the total amount of heat  $q$  received from the combustion is also constrained.

The best way to cool is actually to cool down while spinning up, such that temperature in fact remains constant. A completely isothermal process is not feasible because it would take infinitely long, but, to the extent that it can be reached, it gives the highest efficiency. The process of spinning while cooling is not completely infeasible, because it is done at the ambient temperature, for which a thermal reservoir with infinite heat capacity and  $T_{\text{min}}$  is readily available. Of course, higher efficiency yet would be reached to slow down the spinning also at constant temperature, but for that process there is no heat reservoir with the appropriate temperature.

In summary, the Otto cycle with spinning gas has the following processes: 1. isothermal spinning injection, 2. adiabatic compression, 3. isochoric heating, 4. adiabatic expansion, 5. adiabatic spinning ejection. These processes depend on four dimensionless parameters:  $\delta$ ,  $q$ ,  $J$ , and  $\varphi$ . The efficiency weakly depends on  $J$ ; there are almost no significant changes in varying  $J$  from 0.1 to 10. The efficiency dependence on the other three parameters is shown in Fig. 5, where the dependence of  $\eta$  as a function of  $\varphi$  is plotted for fixed  $\delta$  and  $q$  two.

Two important observations can be made: one, the peak of the efficiency is reached at lower values of  $\varphi$ , likely making it easier to reach in real conditions, since spinning injection might be an issue in real devices; and, two, the maximum value of efficiency is somewhat greater

than estimated by Eq. (8).

*Conclusion* The equilibrium thermodynamic limits of internal combustion engine efficiency is reconsidered with a spinning working gas. Spinning the gas around the axis of the cylinder, while compressing and expanding axially, gives a theoretical efficiency gain of several percent to practical engine cycles, such as the Otto or Diesel cycles. As a practical matter, the spinning might be initiated through compressors or through a flywheel. In arriving

at these conclusions, many of the inefficiencies of real engines were neglected. The new theoretical limit found neglects such non-ideal important effects as friction, insufficient mixing, or heat transfer. However, the increase in the theoretical optimum efficiencies for practical fuel cycles of as much as a few percent is rather remarkable.

*Acknowledgments* This work was supported by DTRA, DOE Contract No. DE-AC02-09CH11466, and by NNSA SSAA Grant No. DE-FG52-08NA28553.

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