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## Finite Element Analysis of Transverse Compressive and Thermal Loads on Nb<sub>3</sub>Sn Wires with Voids

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# Finite Element Analysis of Transverse Compressive and Thermal Loads on Nb<sub>3</sub>Sn Wires with Voids

Y. Zhai, L. d'Hauthuille, C. Barth and C. Senatore

**Abstract**— High field superconducting magnets play an important role in many large-scale physics experiments, particularly particle colliders and fusion confinement devices such as LHC and ITER. The two most common superconductors used in these applications are NbTi and Nb<sub>3</sub>Sn. Nb<sub>3</sub>Sn wires are favored because of their significantly higher J<sub>c</sub> (critical current density) for higher field applications. The main disadvantage of Nb<sub>3</sub>Sn is that the superconducting performance of the wire is highly strain-sensitive and it is very brittle. This strain-sensitivity is strongly influenced by two factors: plasticity and cracked filaments. Cracks are induced by large stress concentrators that can be traced to the presence of voids in the wire. We study the correlation between irreversible strain limit and the void-induced local stress concentrations. We develop an accurate 2D and 3D finite element model containing filaments and different possible distributions of voids in a bronze-route Nb<sub>3</sub>Sn wire. We apply compressive transverse loads for various cases of void distributions to simulate the natural stress and strain response of a Nb<sub>3</sub>Sn wire under the Lorentz force. This study improves our understanding of the effect voids have on the wire's mechanical properties, and in so, the connection between the distribution of voids and performance degradation.

**Index Terms**—Finite element analysis, Nb<sub>3</sub>Sn superconducting wires, fusion magnet, stress concentration.

## I. INTRODUCTION

In the International Thermonuclear Experimental Reactor (ITER), currently under construction in the south of France, two different low-temperature superconductors are used to produce the magnetic fields required to confine the plasma. NbTi is used for the Poloidal Field coil, and Nb<sub>3</sub>Sn is used for the Toroidal Field (TF) coil and Central Solenoid (CS). Nb<sub>3</sub>Sn is used for the TF and CS in tokamaks because of its much higher critical current density for generating higher magnetic field and better confinement from the TF coils and a better current driver with the CS coils.

The two important issues with Nb<sub>3</sub>Sn include that it is very brittle and its critical current density is highly dependent on transverse and axial loadings. It has been found that large enough currents and dimensions cause transverse forces that in turn cause levels of stress above the "irreversible limit", at

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which point the performance of the wire is permanently damaged. The results of an experiment [3] done at the University of Geneva where various loads were applied using a U-shaped groove, with the wire held at 4.2 K and the field varied as shown in Figure 1, which illustrate the aforementioned behavior.

Recent studies have determined that this irreversible limit is due to two phenomena occurring in the wire: cracks and plastic deformation. These two phenomena have extremely similar effects, causing a drop of the wire's critical current density. Recent analysis indicates that the cracks contribute considerable more to the drop in J<sub>c</sub> [3]. The cracked filaments are believed to be a reaction to extreme stress concentrations, which appear to be caused by voids inherent in superconducting Nb<sub>3</sub>Sn. These voids form during the heat treatment process, where Sn in the CuSn matrix that surrounds Nb filaments in the precursor wire reacts with the Nb, forming the superconducting Nb<sub>3</sub>Sn filaments.

Collaboration between University of Geneva and Princeton Plasma Physics Lab is established to understand how certain defects, specifically voids, correlate to this irreversible strain limit which causes the performance degradation. University of

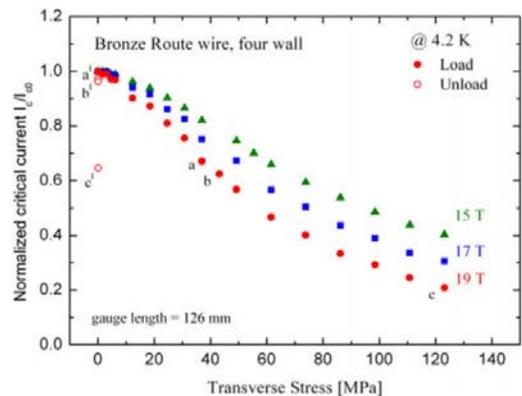


Fig. 1. Transverse stress against critical current of Bronze route wire indicating irreversible stress limit.

Geneva is focusing on tomography experiments with imaging

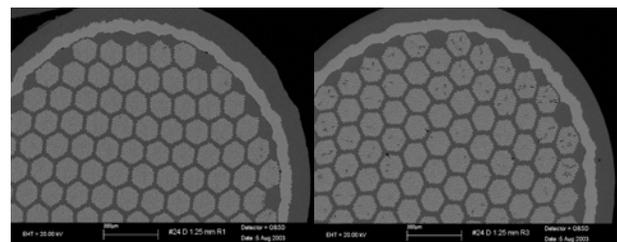


Fig. 2. Bronze-route Nb<sub>3</sub>Sn wire prior to and after heat treatment. on three different types of Nb<sub>3</sub>Sn wires (Powder-in-Tin,

Internal Tin diffusion and Bronze-route), to visualize these voids. Princeton Plasma Physics Lab is performing numerical modeling to analyze how distributions of these voids, along

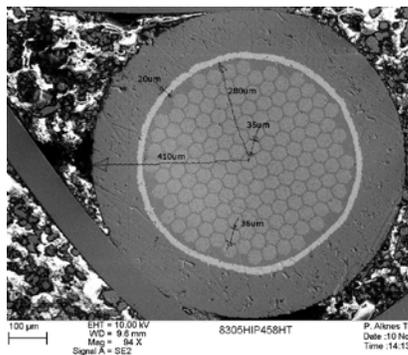


Fig. 3. Bruker Bronze-route  $Nb_3Sn$  wire.

with the shape of voids, cause cracks in the superconducting filaments.

The numerical simulation is performed using finite element

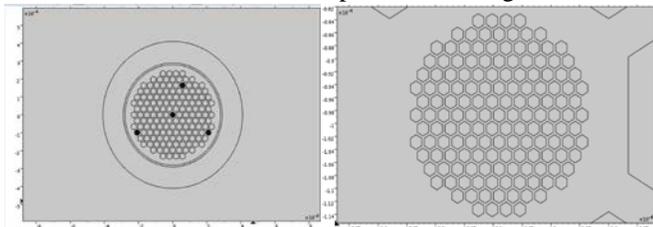


Fig. 4. Two-dimensional CAD model created in COMSOL of wire

models along with fracture mechanics analysis. We built an accurate 2D and 3D finite element model of a Bruker bronze-route  $Nb_3Sn$  wire with details down to the filament level and implement a few distributions of voids, with common shapes, to this model. The 2D model was built primarily to observe perturbations in the stress distribution due to inter-filamentary voids, when transverse loads were applied. The geometry in the 3D model was simpler (not detailed to the filament level) for our simulations, where macroscopic effects were being investigated. The 3D model was used to investigate the pre-strain inherent in a wire at superconducting temperatures, which comes from the cool down from heat treatment temperature (923K) to superconducting magnet operating temperature (4K).

## II. METHODOLOGY

### A. 2D Finite Element Model

The geometry of all the models used in this project is based off of tomographic imaging of a Bruker Bronze-Route  $Nb_3Sn$  superconducting wire (Figure 3) done at the University of Geneva. First, we began by building a very simple 2D CAD model with ideal bundles of  $Nb_3Sn$  as opposed to filaments. As done in experiments, we surrounded the wire by a puck of epoxy, upon which would be applied the boundary conditions and loading. We then add detail to the filament level, which allows for the implementation of voids. Individual filaments are hexagonal and, since their distribution in a bundle is very similar to that of the bundles in the wire itself, we took advantage of this by scaling down the overall geometry of the

bundles and using this to replace some of the bundles with 161 filaments. Many different shaped voids and combinations of voids were also implemented in the geometry, based on shapes seen in the tomographic imaging as well as statistical void data from the X-ray tomography experiment.

The material properties in a wire composite, i.e. Epoxy, Copper, Tantalum, Bronze,  $Nb_3Sn$ , were assumed to be at 4 K and perfectly elastic. Predominantly, these were based on [1], although the Young's modulus and Poisson's ratio of the epoxy casting came from [2]. The voids were modeled as vacuum (empty space with free boundaries).

The discretization of the model is done using COMSOL's adaptive meshing feature. This aids significantly in optimizing the speed without sacrificing accuracy and resolution, by meshing into extremely small elements in and near smaller domains, such as filaments and voids, and larger elements in places like the outer copper shell, where there is a large amount of the same material experiencing similar conditions. There were a total of  $\sim 90,000$  elements in our model when it contained voids.

As for boundary conditions, since for our problem, the wire is extremely long compared to its diameter and the loads are acting perpendicular to its length, we can apply the plane

TABLE I  
UNITS FOR MAGNETIC PROPERTIES (SHORT TITLE HERE IN SMALL CAPS)

Materials	Young's Modulus (GPa)	Poisson's Ratio
Copper	137	0.35
Tantalum	187.8	0.34
Bronze	137	0.34
$Nb_3Sn$	100	0.3
Epoxy	8	0.3

strain approximation for our problem by assuming the strain along the length is zero. Define our axes such that length is parallel to the  $z$  direction.

We apply boundary conditions that simulate the loads that a wire would experience during runtime. For 2D study, we keep the left and right sides of the epoxy casting free, while fixing the bottom boundary and applying a force uniformly across the top boundary. Different forces were experimented, however the results in this paper will focus primarily on a load value of 17 kN. This value is based on [2], where it was experimentally found that applying this amount of force caused a significant permanent degradation in the critical current density upon unloading. On one hand, this value is somewhat of a magic number as UNIGE's experiment was done with similarly sized, Powder-In-Tin wires, instead of the bronze-route wires that our FEA is based on. Since our study focuses on the correlation between void presence and distribution on stress concentrations due to loads, the actual value itself is not significant.

Finally, we run the FEA through COMSOL and post-processing will consist primarily of looking at Von Mises stress plots. These can be looked at macroscopically by using color plots of various ranges, but we can also obtain stress values at specific points on the plot, which will help pinpoint the peak stresses caused by voids, both in the bronze matrix as

well as in an individual filament.

### B. 3D Finite Element Model

For our 3D model, the main difference lies in the material properties and loading conditions. Since we use 3D model attempt to understand the pre-strain in the wire after a cool down from heat treatment temperature (923K) to magnet operating temperature (4K), properties must now be

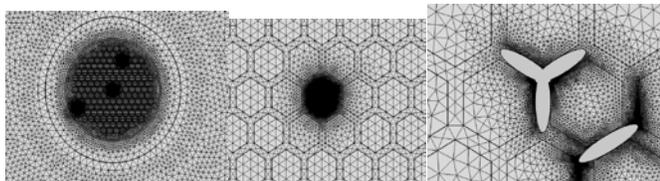


Fig. 5. Images of the mesh. The left is zoomed in close to bundles – the center is a set of filaments, while the right is zoomed in close to voids – the hexagons seen are instead individual filaments.

temperature dependent. The geometry is created by extruding the 2D model with the idealized bundles of  $Nb_3Sn$ .

The material properties for this 3D model are again based on [1]. It is assumed, based on expected thermal stress loads, that only Tantalum, Copper and Bronze become plastic. Mitchell derives the Young's Moduli and coefficients of thermal expansion using a 1D finite element model for all materials. Since we know that Copper, Tantalum and Bronze all undergo plastic deformation during the cool down process, our model must deal with this. Instead of using a non-linear stress strain curve, we will use a bilinear stress-strain model. The yield stress for the bilinear model will come directly from Mitchell's work, and determining the Tangent modulus is done by assuming a yield strain of 0.1% and computing the slope of the stress-strain curve between this yield strain and a strain of 1%. 1% is used as a maximum since this is the maximum amount of strain that copper, the most ductile material, would encounter during the process. A significantly coarser adaptive mesh was used, since performing FEA for a 3D model is extremely computationally intensive.

One end of the wire was held fixed, and a thermal load was applied evenly across the whole wire. The load to be modeled

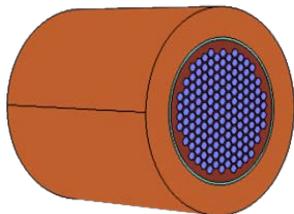


Fig. 6. The 3D CAD geometry

here was one where the entire wire experienced a linearly changing temperature (from 923K to 4K) over time. This changing temperature was implemented across all domains of the wire in COMSOL. The thermal strain at different times, and thus different temperatures, was then investigated using a color plot. This strain was then compared to our theoretical expectations.

## III. RESULTS AND ANALYSIS

### A. 2D Finite Element Model

Prior to the implementation of voids, we applied the aforementioned load conditions on the 2D model of the wire, with four of the bundles replaced with sets of filaments. It was found that, based on the average stress in the bundles, filaments and the bronze between filaments (Table II), our simplified model is accurate, that is, it is not necessary to replace every bundle in the wire with sets of filaments. This is an important preliminary test because it proves to us that this model we will be using is accurate.

It was also found that, globally, Von Mises stress in a

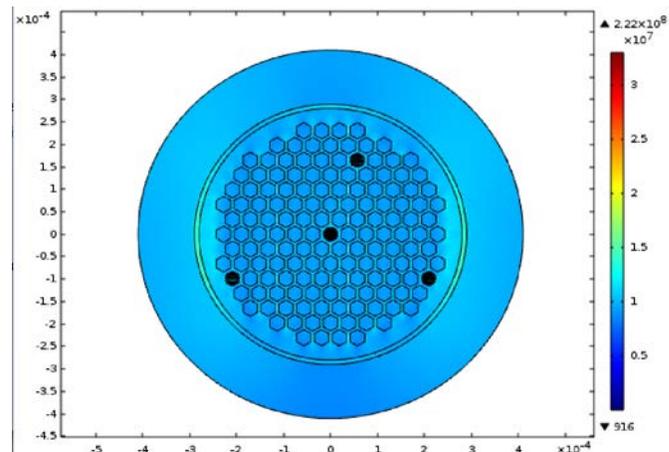


Fig. 7. Von Mises stress of model without voids vs. model with voids

model with voids looks almost identical to a model without voids, suggesting that the stress concentrations caused by voids have a very small range of effect. This result is depicted

TABLE II  
UNITS FOR MAGNETIC PROPERTIES (SHORT TITLE HERE IN SMALL CAPS)

Symbol	Bundle	in filaments	in filaments
Mean Stress (MPa)	9.3	9.5	10.36

in Figure 7.

Locally, on the other hand, voids cause great perturbations in the stress distribution. Concentrations of high stress appear near voids and interestingly, a significant decrease in stress also occurs around parts of the voids. This odd decrease in stress can be attributed to the fact that the stress flux lines are

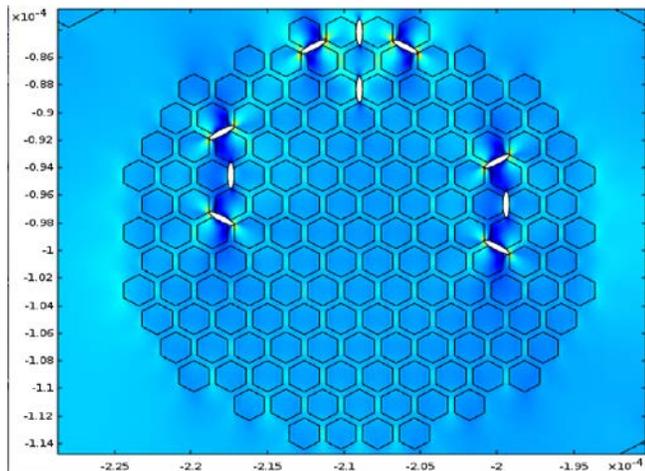


Fig. 8. Von Mises stress of bottom left set of filaments.

pulled to areas of high stress concentration, leaving nearby areas with much lower density of stress flux lines than would be there were there no voids. Figures 8-10 show various distributions of voids as well as different shapes of voids, so their effects can be qualitatively seen. Approximate values for the peak and minimum stress in the matrix, as well as the peak stress in the filament due to various voids shapes can be found in the Table III.

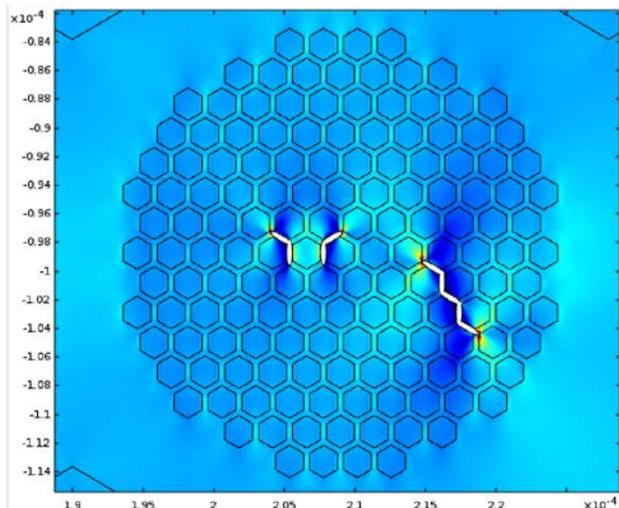


Fig. 9. Von Mises stress of bottom right set of filaments.

### B. 3D Finite Element Model

The results from solving the 3D model for the thermal loading that occurs during the volumetric cool down from 923 K to 4 K can be seen in the stress plots of three slices of the wire at temperatures 693 K, 464 K and 234 K (Figure 10). The thermal stress progresses as expected for the cool down, except that we can see that as it reaches 234 K, an unexpectedly large amount of stress appears in 2 areas of the fixed end. This is due to an inverted mesh error occurring somewhere around 250 K, due to a large deformation compared to the size of the mesh element. This corrupts the

rest of the results, and does not allow for an accurate analysis to 4 K.

TABLE III  
PEAK AND MINIMUM STRESS IN THE MATRIX NEAR VOID AND PEAK STRESS OF CLOSEST FILAMENT

Void Type	Ellipse, 30/150	Ellipse, 90	Long Crack	Short Crack	Circular (A/B/C)
Peak stress (MPa)	93.2/92.9	20.7	219.2	143.4	143.4/31.3/44.6
Min stress (MPa)	0.1/0.17				1.69/0.58/1.9
Filament	61.5/65.5	17.6	154.6	114.1	114.08/20.1/16.1/3



Fig. 10. Von Mises stress of these slices at temperature

The thermal strain of a slice of the wire at 464 K agrees with theoretical predictions for the thermal strain for the materials, thus demonstrating that our model is accurate for the cool down up to that temperature.

### IV. CONCLUSION

An accurate model of a bronze-route  $Nb_3Sn$  wire, detailed to the filament level was built, inside which voids were implemented in a realistic manner. The studies have given some important insight on the effect of voids on stress distributions. Specifically, it has been found that both shape and orientation play an important role in stress concentration, as shown in Table III. For example, the placement of circular or vertical, elliptical voids can cause a peak stress in the nearest filament that is only about a factor 1.5 larger than the mean stress without voids. One would expect these types of voids to be insignificant in the cracking of filaments. However, a long "crack" type void has been shown to cause peak stresses in the filaments that are a factor of 15 larger than the mean stress in filaments, which can be a significant cause for cracking.

It has also been found that the range of effect of voids is local. While the effects of nearby voids can and do stack, the range of effect is small enough that it will not extend past the set of filaments in which they are located. This explains why globally, the stress profile of a wire with a realistic amount of voids and without voids looks almost indistinguishable. How large the range of effect of voids is has not yet been quantified, but is certainly shape, size and orientation dependent.

A 3D bundle model was also developed. The essential temperature dependent material properties, i.e. coefficient of thermal expansion, Young's modulus, Poisson's ratio, yield stress and the tangent modulus were implemented based on [1]. Poisson's ratio was assumed to remain constant, as the literature suggests, and a bilinear stress-strain model was created for materials that were expected to become plastic, in lieu of their non-linear stress-strain curves. For this bilinear model, the tangent modulus for each plastically inclined

material was derived for a range of 0.1% to 1% from the aforementioned curves.

A thorough analysis of the cool down from the heat treatment temperature of 923 K to the working temperature of 4 K is essential for future analysis of the effect of voids on stress during mechanical loading in a Nb<sub>3</sub>Sn wire because significant accumulation of thermal pre-strain, which has non-negligible effects the mechanical properties of the wire. The 3D model appears to be functioning as expected during the cool down, based on the thermal strain present throughout the wire when half of the cool down is completed. However, to complete the analysis, the inverted mesh error must be rectified. It is postulated that this error is perhaps due to too large of a time step or potentially, the cusp in the bilinear model.

The 3D statistical information of voids from the experimental tomography is available and this data will allow, in conjunction with the knowledge gained from our analysis of the 2D and 3D finite element models, a full analysis of the effect of voids acquired during the heat treatment process on the mechanical properties of the wire and, using fracture mechanics, how the stress concentrations caused by the voids lead to the cracking of filaments.

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