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Magnetohydrodynamics for Collisionless Plasmas from the Gyrokinetic Perspective

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Abstract

The effort to obtain a set of hydromagnetic equations for a magnetized collisionless plasma started nearly 60 years ago by Chew, Goldberger and Lowe. Many attempts have been made ever since. Here, we will show the derivation of a set of collisionless MHD equations from the gyrokinetic perspective. This set of equations is energy conserving and, in the absence of fluctuations, recovers the usual MHD equilibrium. Furthermore, the corresponding plasma pressure balance can be modified by the finite-Larmor-radius (FLR) effects in the regions with steep pressure gradients. The present work is an outgrowth of the paper on "Alfven Waves in Gyrokinetic Plasmas" by W. W. Lee and H. Qin [Phys. Plasmas **10**, 3196 (2003)].

The search for a set of one-fluid hydromagnetic equations for collisionless plasmas from the Boltzmann equation started sixty years ago by Chew, Goldberger and Low (CGL) [1]. This attempt was made because the usual fluid equations were derived from collisional considerations [2]. The CGL work was followed by Kulsrud [3] as well as Frieman, Davidson and Langdon [4]. However, the work on this interesting subject has not received much attention over the years, since the use of ideal MHD, based on usual MHD equations, applied to collisionless plasmas, has been proven empirically to be very useful. In the present paper, we will show the derivation of a set of collisionless MHD equations by applying the gyrokinetic ordering [5] on the Vlasov-Maxwell equations. The main ingredient of this connection is the contribution of the ion polarization drift on the quasineutality condition [6], which we will explain. An initial attempt based on this methodology was made more than ten years ago [7].

Let us first re-visit the subject of the gyrokinetic approximation,

$$\omega/\Omega \sim (k_\perp \rho_i) e(\phi - \mathbf{v}\cdot\mathbf{A})/T_e \sim k_\parallel \rho_i \sim o(\epsilon),$$

for the Vlasov-Maxwell equations [5, 8], where ω is the frequency of interest, Ω is the cyclotron frequency, ϕ and $\mathbf{A}$ are the perturbed electrostatic and vector potentials, respectively, $k_\parallel$ and $k_\perp$

are the wave vectors parallel and perpendicular to the zeroth-order magnetic field, respectively, and ϵ is a smallness parameter. The paper is closely related to that of Lee and Qin [7], but involves a new way of deriving the governing gyrokinetic equations as well as the new physics insight in terms of gyrokinetic MHD equations arising from finite-Larmor-radius (FLR) effects.

The governing gyrokinetic Vlasov-Maxwell equations used in the present paper can be derived by first changing the original Vlasov equation,

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial F_\alpha}{\partial \mathbf{x}} + \frac{q}{m}\left[\mathbf{E} + \frac{1}{c}\mathbf{v}\times(\mathbf{B}_0 + \delta\mathbf{B})\right]\cdot\frac{\partial F_\alpha}{\partial \mathbf{v}} = 0, \tag{1}$$

where $F_\alpha \equiv F_\alpha(\mathbf{x},\mathbf{v},t)$ is the distribution function in six dimensional phase space, α denotes species,

$$\mathbf{E} = -\nabla\phi - (1/c)\partial\mathbf{A}/\partial t,$$

$$\delta\mathbf{B} = \nabla\times\mathbf{A},$$

and $\mathbf{B}_0$ is the equilibrium background magnetic field. Making use of the Lagrangian,

$$L = \frac{1}{2}mv^2 - q\phi + \frac{q}{c}\mathbf{v}\cdot\mathbf{A}.$$

as described, for example, by Corben and Stahle [9], we then obtain

$$\frac{\partial F_\alpha}{\partial t} + (\mathbf{v} + \frac{q_\alpha\mathbf{A}}{m_\alpha c})\cdot\frac{\partial F_\alpha}{\partial \mathbf{x}} + \frac{q}{m}\left[-\nabla(\phi - \frac{1}{c}\mathbf{v}\cdot\mathbf{A}) + \frac{1}{c}\mathbf{v}\times\mathbf{B}_0\right]\cdot\frac{\partial F_\alpha}{\partial(\mathbf{v} + q_\alpha\mathbf{A}/m_\alpha c)} = 0,$$

where ϕ and $\mathbf{A}$ are the perturbed scalar and vector potentials, respectively. Alternatively, by changing the phase variables, we can re-write the equation as

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial F_\alpha}{\partial \mathbf{x}} + \frac{q}{m}\left[-\nabla(\phi - \frac{1}{c}\mathbf{v}_\perp\cdot\mathbf{A}_\perp) - \frac{1}{c}\frac{\partial\mathbf{A}_\parallel}{\partial t} + \frac{1}{c}\mathbf{v}\times(\mathbf{B}_0 + \delta\mathbf{B}_\perp)\right]\cdot\frac{\partial F_\alpha}{\partial(\mathbf{v} + q_\alpha\mathbf{A}_\perp/m_\alpha c)} = 0,$$

where the subscripts $\parallel$ and $\perp$ denote the direction parallel and perpendicular to $\mathbf{B}_0$, respectively, and the approximation of

$$\mathbf{v}_\perp + \frac{q_\alpha\mathbf{A}_\perp}{m_\alpha c} \approx \mathbf{v}_\perp,$$

for $q_\alpha\phi/T_\alpha \sim q_\alpha v_\perp A_\perp/cT_\alpha \ll 1$ is used.

To derive the gyrokinetic Vlasov-Maxwell equations, one could follow the procedures used in Ref. [6] or, more formally, those of Ref. [10] based on the Lie transform and a non-canonical perturbation theory. For the present purpose, we derive them using the simplified method of Ref. [7] by applying the drift kinetic approximation for the velocities associated with the $\mathbf{v}\cdot\nabla$ and

$\mathbf{v} \times (\mathbf{B}_0 + \delta\mathbf{B}_\perp)$ terms in the above equation. The corresponding guiding center approximation becomes

$$\mathbf{v} \approx v_\parallel \mathbf{b} + \frac{c}{B_0}\mathbf{E} \times \mathbf{b}, \tag{2}$$

where

$$\mathbf{E} = -\nabla(\phi - \mathbf{v}_\perp \cdot \mathbf{A}_\perp/c) - (1/c)\partial\mathbf{A}_\parallel/\partial t,$$

$$\mathbf{b} = \hat{\mathbf{b}}_0 + \delta\mathbf{B}_\perp/B_0,$$

$$\hat{\mathbf{b}}_0 = \mathbf{B}_0/B_0,$$

and

$$\delta\mathbf{B}_\perp = \nabla \times \mathbf{A}_\parallel.$$

This drift kinetic approximation for the velocities is consistent with the formulations given in Refs. [6, 10]. It can then be shown that the governing gyrokinetic equations based on the Darwin approximation [7, 11], including both scaler potential, ϕ, and vector potentials, $\mathbf{A}_\parallel$ and $\mathbf{A}_\perp$, in slab geometry take the form of

$$\frac{\partial F_\alpha}{\partial t} + \left[v_\parallel \mathbf{b} - \frac{c}{B_0}\nabla(\bar{\phi} - \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}/c) \times \hat{\mathbf{b}}_0\right] \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} - \frac{q}{m}\left[\nabla(\bar{\phi} - \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}/c) \cdot \mathbf{b} + \frac{1}{c}\frac{\partial \bar{A}_\parallel}{\partial t}\right]\frac{\partial F_\alpha}{\partial v_\parallel} = 0, \tag{3}$$

where

$$\nabla^2\phi + \frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp^2\phi = -4\pi\sum_\alpha q_\alpha \int \bar{F}_\alpha dv_\parallel d\mu, \tag{4}$$

$$\nabla^2\mathbf{A} - \frac{1}{v_A^2}\frac{\partial \mathbf{A}_\perp}{\partial t^2} = -\frac{4\pi}{c}\sum_\alpha q_\alpha \int \mathbf{v}\bar{F}_\alpha dv_\parallel d\mu, \tag{5}$$

$$\mu \equiv v_\perp^2/2 \approx const.,$$

and the $\overline{bar}$ quantities denote gyrophase averages. The derivations of the gyrokinetic Poisson's equation, Eq. (4), and Ampere's law, Eq. (5), based on the longitudinal ion polarization drift of

$$\mathbf{v}_p^L = -(m_i c^2/eB^2)(\partial\nabla_\perp\phi/\partial t)$$

and the transverse ion polarization drift of

$$\mathbf{v}_p^T = -(m_i c/eB^2)(\partial^2\mathbf{A}_\perp/\partial^2 t),$$

respectively, can be found, for example, in Ref. [7]. These additional terms associated with the density and current responses are the result of the gyrokinetic approxmation for the distribution function, F_α, in Eq. (1), which brakes up to three different parts as given by Eqs. (3), (4) and (5), respectively. Equation (3) is in agreement with the slab version of the equation used in Refs. [12] and [13]. Assuming F_α in Eq. (3) is independent of the gyrophase angle associated with the rotation of $\mathbf{v}_\perp$, based on the gyrokinetic ordering argument, the gyrophase-averaged quantity of $\mathbf{v}_\perp \cdot \mathbf{A}_\perp$, becomes [12]

$$\overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} = -\frac{1}{2\pi}\frac{eB_0}{mc}\int_0^{2\pi}\int_0^{\rho} \delta B_\| r dr d\theta,$$

where $\rho = v_\perp/\Omega$ and $\Omega = eB/mc$. The energy conservation of the system becomes

$$\frac{d}{dt}\left\langle \int (\frac{1}{2}v_\|^2 + \mu)(m_e F_e + m_i F_i) dv_\| d\mu + \frac{\omega_{ci}^2}{\Omega_i^2}\frac{|\nabla_\perp \Phi|^2}{8\pi} + \frac{|\nabla A_\||^2}{8\pi} \right\rangle_{\mathbf{x}} = 0, \tag{6}$$

where $\Phi \equiv \phi - \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}/c$ and $\langle \cdots \rangle_{\mathbf{x}}$ denotes spatial average. Here, the approximation of

$$\mathbf{v}\cdot\mathbf{v} \approx \mathbf{v}_\| \cdot \mathbf{v}_\| + \mathbf{v}_\perp \cdot (\mathbf{v}_\perp + 2\frac{q_\alpha}{m_\alpha c}\mathbf{A}_\perp)$$

has been used to calculate the particle kinetic energy. Thus, the energy conservation to the quadratic order in the perturbed potentials are independent of $\mathbf{A}_\perp$, where $\delta \mathbf{B}_\| \approx \nabla_\perp \times \mathbf{A}_\perp$ for $k_\| \ll k_\perp$. For comparison, using the guiding-center approximation of Eq. (2), we obtain, from Eq. (1), the governing drift kinetic equation as

$$\frac{\partial F_\alpha}{\partial t} + \left[v_\| \mathbf{b} - \frac{c}{B_0}(\nabla\phi + \frac{1}{c}\frac{\partial \mathbf{A}_\perp}{\partial t}) \times \hat{\mathbf{b}}_0 \right] \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} - \frac{q}{m}\left[\nabla\phi \cdot \mathbf{b} + \frac{1}{c}\frac{\partial A_\|}{\partial t} \right] \frac{\partial F_\alpha}{\partial v_\|} = 0,$$

which gives the same energy conservation, Eq. (6), in the low frequency limit. We prefer the formulation given by Eq. (3) here, since, in the limit of $\rho_i \to 0$, we can simply argue that the system is independent of $\mathbf{A}_\perp$. As one can see, in the limit of $\rho \to 0$, $\mathbf{A}_\perp$ does not appear in Eq. (3) together with $\bar{\phi} \to \phi$, $\bar{A}_\| \to A_\|$, and $\bar{F} \to F$, respectively, they then become the starting equations in Ref. [14].

For the general toroidal geometry, the gyrokinetic Vlasov equation can be re-written as, e.g. Ref. [15],

$$\frac{\partial F_\alpha}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial F_\alpha}{\partial \mathbf{R}} + \frac{dv_\|}{dt}\frac{\partial F_\alpha}{\partial v_\|} = 0, \tag{7}$$

$$\frac{d\mathbf{R}}{dt} = v_\| \mathbf{b}^* + \frac{v_\perp^2}{2\Omega_{\alpha 0}}\hat{\mathbf{b}}_0 \times \nabla ln B_0 - \frac{c}{B_0}\nabla\bar{\Phi} \times \hat{\mathbf{b}}_0,$$

and

$$\frac{dv_\|}{dt} = -\frac{v_\perp^2}{2}\mathbf{b}^* \cdot \nabla ln B_0 - \frac{q_\alpha}{m_\alpha}\left(\mathbf{b}^* \cdot \nabla\bar{\Phi} + \frac{1}{c}\frac{\partial \bar{A}_\|}{\partial t} \right),$$

where

$$F_\alpha = \sum_{j=1}^{N_\alpha} \delta(\mathbf{R} - \mathbf{R}_{\alpha j})\delta(\mu - \mu_{\alpha j})\delta(v_\parallel - v_{\parallel\alpha j}),$$

$\Omega_{\alpha 0} \equiv q_\alpha B_0/m_\alpha c$, $\mathbf{b}^* \equiv \mathbf{b} + (v_\parallel/\Omega_{\alpha 0})\hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla)\hat{\mathbf{b}}_0$, and $\mathbf{b} = \hat{\mathbf{b}}_0 + \nabla \times \bar{\mathbf{A}}/B_0$. Again, the variables with subscript "0" represent equilibrium quantities.

Now let's look at the gyrokinetic current density for the gyrocenters. For $k_\perp \rho_i \sim 1$, we have

$$\mathbf{J}(\mathbf{x}) = \mathbf{J}_\parallel(\mathbf{x}) + \mathbf{J}_\perp^M(\mathbf{x}) + \mathbf{J}_\perp^d(\mathbf{x})$$

$$= \sum_\alpha q_\alpha \langle \int F_\alpha(\mathbf{R})(\mathbf{v}_\parallel + \mathbf{v}_\perp + \mathbf{v}_d)\delta(\mathbf{R} - \mathbf{x} + \rho)d\mathbf{R}dv_\parallel d\mu\rangle_\varphi,$$

where

$$\mathbf{v}_d = \frac{v_\parallel^2}{\Omega_\alpha}\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}})\hat{\mathbf{b}} + \frac{v_\perp^2}{2\Omega_\alpha}\hat{\mathbf{b}} \times \frac{\partial}{\partial \mathbf{R}} lnB.$$

For $k_\perp \rho_i \ll 1$, they can be written as [7]

$$\mathbf{J}_\perp^M(\mathbf{x}) = -\sum_\alpha \nabla_\perp \times \frac{c\hat{\mathbf{b}}}{B} p_{\alpha\perp}$$

and

$$\mathbf{J}_\perp^d = \frac{c}{B}\sum_\alpha \left[p_{\alpha\parallel}(\nabla \times \hat{\mathbf{b}})_\perp + p_{\alpha\perp}\hat{\mathbf{b}} \times (\nabla lnB)\right],$$

where

$$p_{\alpha\perp} = m_\alpha \int (v_\perp^2/2)F_\alpha(\mathbf{x})dv_\parallel d\mu,$$

and

$$p_{\alpha\parallel} = m_\alpha \int v_\parallel^2 F_\alpha(\mathbf{x})dv_\parallel d\mu.$$

Now, the current density takes the form of

$$\mathbf{J}_\perp = \mathbf{J}_\perp^M + \mathbf{J}_\perp^d$$

$$= \frac{c}{B}\sum_\alpha \left[\hat{\mathbf{b}} \times \nabla p_{\alpha\perp} + (p_{\alpha\parallel} - p_{\alpha\perp})(\nabla \times \hat{\mathbf{b}})_\perp\right]$$

$$\approx \frac{c}{B}\sum_\alpha \hat{\mathbf{b}} \times \nabla p_\alpha$$

for $p_\alpha = p_{\alpha\parallel} \approx p_{\alpha\perp}$.

With the gyrokinetic Poisson's equation of

$$\frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp^2\phi = -4\pi\rho \tag{8}$$

and the parallel Ampere's law for the electrons of

$$\nabla^2 A_\parallel = -\frac{4\pi}{c} J_\parallel \tag{9}$$

by ignoring $\mathbf{A}_\perp$, we obtain a simple set of gyrokinetic MHD equations for collisionless plasmas with

$$\mathbf{J}_\perp = \frac{c}{B}\hat{\mathbf{b}} \times \nabla p \tag{10}$$

as the current density associated for a given pressure profile, where $p \approx p_i$ with

$$\frac{d}{dt}\nabla_\perp^2 \phi - 4\pi \frac{v_A^2}{c^2} \nabla \cdot (\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0 \tag{11}$$

as the vorticity equation, which can be obtained from Eqs. (7), (8) and (9), together with parallel Ohm's law of the form

$$E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b} \cdot \nabla \phi \approx -\frac{T_e}{e}\frac{1}{p_e}\frac{\partial p_e}{\partial x_\parallel} \rightarrow 0, \tag{12}$$

which can be derived by using Eqs. (3) and (5) as shown earlier by Ref. [3], as well as the incompressible adiabatic equation of state of

$$\frac{dp}{dt} = 0, \tag{13}$$

implying that the energy and mass convect together, where

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi \times \mathbf{b} \cdot \nabla,$$

and $p_e = n_e T_e$ for the electrons. With $\mathbf{J}_\parallel$ given by Eq. (9), Eq. (10) - (13) are the governing gyrokinetic MHD equations, which conserve energy, for $E_\parallel \rightarrow 0$, as

$$\frac{\partial}{\partial t}\int \frac{1}{8\pi}\left(|\nabla_\perp \phi|^2 + \frac{v_A^2}{c^2}|\nabla A_\parallel|^2\right) d\mathbf{x} = -\frac{v_A^2}{c^2}\int \mathbf{E}_\perp \cdot \mathbf{J}_\perp d\mathbf{x},$$

and reduce to MHD equilibrium, when $\phi \rightarrow 0$, as

$$\nabla \cdot (\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0. \tag{14}$$

Thus, we have finally accomplished what Strauss [16] and [17] set out to do and beyond, but without using the aspect ratio (a-minor radius/R-major radius) expansion for a tokamak. It should be noted that the pioneering work by Strauss using the fluid approach mentioned here has inspired many researchers in this area for years. The difference here is that our approach is purely kinetic in nature similar to those of Chew-Goldgerger-Low [1], Kulsrud [3] and Frieman-Davidson-Langdon [4]. Note that Eq. (11) now includes the term, which was absent in Eq. (32) of Lee and Qin [7], and, in turn, gives us the MHD equilibrium of Eq. (14).

Another interesting aspect of finite Larmor radius gyrokinetics is the existence of the equilibrium zonal flows associated with zeroth-order inhomogeneity. Lets us elaborate. As first pointed out by Lee [6], the zeroth-order inhomogeneity also contributes to an extra ion particle density in addition to the ion gyrocentrer density as given by Eq. (40) in that paper. A more complete expression is given by Eq. (17) in Ref. [18], as well as those in Ref. [19], and can be written as

$$\frac{n_i|_{particle}}{n_i} = 1 + \frac{1}{2}\rho_i^2 \frac{1}{p_i}\nabla_\perp^2 p_i,$$

where $p_i \equiv n_i T_i$ and $\rho_i \equiv v_{ti}/\Omega_i$ is the ion gyroradius and v_{ti} is the ion thermal velocity. From the gyrokinetic Poisson's equation, Eq. (8), it gives rise to an equilibrium $\mathbf{E}\times\mathbf{B}$ velocity of

$$\mathbf{v}_{E\times B} \approx -\frac{1}{2}\frac{\nabla_\perp p_i}{p_i}\frac{cT_i}{eB}\hat{\mathbf{b}}\times\hat{\mathbf{x}},$$

where $\mathbf{x}$ is the direction of the zeroth-order inhomogeneity. The corresponding current is given by

$$\mathbf{J}_\perp^{E\times B}(\mathbf{x}) = \sum_\alpha q_\alpha \langle \int \mathbf{v}_{E\times B}(\mathbf{R}) F_\alpha(\mathbf{R})\delta(\mathbf{R}-\mathbf{x}+\rho) d\mathbf{R} d\mu dv_\parallel \rangle_\varphi.$$

Consequently, by taking into account the difference between the electrons and the ions for the $\mathbf{E}\times\mathbf{B}$ drift due to the finite Larmor radius effects, we obtain a new pressure balance equation modified by the current associated with the equilibrium zonal flows as

$$\mathbf{J}_\perp = \frac{c}{B}\hat{\mathbf{b}}\times\nabla p + en_i\frac{\rho_i^2}{2}\left[\nabla_\perp^2 \mathbf{v}_{E\times B} + \frac{\mathbf{v}_{E\times B}}{p_i}\nabla_\perp^2 p_i\right],$$

which can then be simplified to

$$\mathbf{J}_\perp \approx \frac{c}{B}\hat{\mathbf{b}}\times\nabla p\left[1 - \frac{1}{4}\rho_i^2\frac{\nabla_\perp^2 p}{p}\right]. \tag{15}$$

by assuming that $\nabla_\perp^2 \mathbf{v}_{E\times B} \approx 0$ and letting $p_i \equiv p$. Thus, Eq. (15) should be used instead of Eq. (10) in the regions with steep pressure gradient. The corresponding plasma pressure balance from Ampere's law now becomes

$$\nabla\left[\frac{B^2}{8\pi} + p\left(1 - \frac{1}{4}\rho_i^2\frac{\nabla_\perp^2 p}{p}\right)\right] = 0.$$

Thus, we have shown in this paper, that the Darwin gyrokinetic model of Eqs. (3), (4) and (5) can indeed be reduced to a set of MHD equations in the collisionless limit, a pursuit started sixty years ago based on the Vlasov-Maxwell equations [1]. The key to such a connection is the presence of the ion polarization density density in the gyrokinetic Poisson's equation, Eqs. (4) and (8), which was first identified by Lee [6]. Not surprisingly, this set of equations are different from the conventional MHD equations, notably the absence of the compressional Alfven waves, which can be ignored through the ordering argument with the present formulation. Nevertheless, it can still recover the conventional MHD equilibrium, Eq. (14). Furthermore, the present paper also points out the corrections to these equations due to FLR effects.

In the future, it would be interesting to include the higher order fluid moments, such as heat fluxes and etc., as well as the compressional components of the Alfven waves in these gyrokinetic MHD equations. Moreover, the connection between these gyrokinetic equations and the MHD equilibria as shown in the present paper suggests that it is feasible to devise an iterative scheme between a gyrokinetic code and an MHD equilibrium code with the purpose of minimizing turbulence and anomalous transport in tokamaks based on an iterative procedure, which first decouples the transport problem from the equilibrium problem, and then couples them through global parameter exchanges [20].

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